Institut d'Etudes Politiques de Paris

ECOLE DOCTORALE DE SCIENCES PO

Programme doctoral en économie

Doctorat en Sciences économiques

Bubbles and Asset Supply

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Soutenue à Paris le 2 juillet 2013

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A Fabien et Laurent,
et à mes parents, qui ont voulu tout donner à leurs enfants.

Abstract

The first chapter of the thesis makes the point that capital is not scarce any more in advanced economies. Compared to previous work, I update data on mixed income and land rents, and I find that the criterion for dynamic efficiency is not verified for any advanced economy ; and that Japan and South Korea have unambiguously over-accumulated capital. Subject to some caveats, an increase of public debt, or a generalization of pay-as-you-go systems could therefore be Pareto-improving. The consequences of this world "savings glut" for positive analysis are important. It can potentially help to explain a number of macroeconomic stylized facts like low interest rates and financial (rational) speculative bubbles.

In the second chapter of this thesis, I show that leverage and short-sales constraints are crucial in understanding the formation and the location of rational speculative bubbles. I study adaptive learning of asset prices when priors initially differ because of the multiplicity of rational expectations equilibria when economies are dynamically inefficient. The availability of leverage and short-sales strongly affects asset prices and beliefs on the learning path : in particular, leverage can increase the amount of "overoptimism" in the economy, and feed bubbles' rise. Short-sales can have an opposite effect. In the long run, optimism survives if the economy is dynamically inefficient. In that case, adaptive learning provides a selection criterion for the location of rational bubbles, and a mechanism by which they can grow faster than the rate of interest in a non-stochastic environment. The model also can explain why bubbles usually deflate over weeks sometimes months rather than burst instantaneously. It generates countercyclical margins and procyclical leverage without the assumption of "scary bad news", return predictability, and equilibrium default. Empirically, speculative bubbles often form in environments of so-called financial deregulation, which helps leverage. The model I develop helps think about how to make public debt assets safer.

In the third chapter, I show that rational speculative bubbles can drive the business cycle. For that matter, I augment Peter Diamond's life cycle neoclassical growth model with a labor/leisure margin and rational bubbles shocks. I show that those shocks can drive business cycles fluctuations, leading to higher consumption, investment and hours. Elasticities can be calibrated to be compatible with balanced growth, and microeconomic estimates for labor supply, while generating large fluctuations in hours. The model features countercyclical labor wedges even though wages are set competitively, that come from the decrease of interest rates during bubbles' busts. It can generate "jobless recoveries": the stock market can rebound with initially decreasing levels of employment.

In the fourth chapter coauthored with Thomas Grjebine, we study the causal link between house prices and current accounts. Across time and countries, we find a very large and

significant impact of house prices on current accounts. In order to rule out endogeneity concerns, we instrument house prices for a panel of countries, using property tax variations. A 10% instrumented appreciation in house prices leads to a deterioration in the current account of 1.7% of GDP. These results are very robust to the inclusion of the determinants of current accounts. These results seem to suggest that asset overvaluations could drive international business cycles as well.

Finally, the fifth chapter is more normative and is a first step towards thinking about capital taxation in a world with very abundant savings. The conclusions one draws from this analysis is far from clear-cut. Life cycle models of capital accumulation with land (or monopoly rents or decreasing returns) can only feature capital under accumulation, because of the extreme capital crowding out properties of capitalizable rents as one approaches the Golden Rule. I show that it is no longer valid when a government levies property, wealth or estate/gift taxes. Contrary to infinite horizon models, positive capital taxes therefore help capital accumulation towards the Golden Rule in overlapping-generations models ; and no other set of transfers from young or old, public debt, or social security system can help achieve such a level of capital accumulation. On the other hand, more capital taxation would certainly end up depressing accumulation at a certain point.

Contents

Page

Introduction **1**

1 Reassessing Dynamic Efficiency **7**

1 Sufficient conditions for dynamic efficiency 11
2 Reassessing dynamic efficiency 12
3 Further calculations 15
4 Discussion 20

2 Bubbles and Adaptive Learning **41**

1 The model 44
2 Static determination of asset prices 46
3 Econometric learning 54
4 Constant gain learning 60
5 Policy measures 63
6 Conclusion 66
Appendices 70

3 Rational Bubbles Driven Business Cycles **81**

1 The model 85
2 Labor supply elasticities: A continuous time life cycle model 87
3 Steady-states 91
4 Shocks 93
5 Conclusion 94
Appendices 99

4 House Prices Drive Current Accounts: Evidence from Property Tax Variations **111**

1 Data and estimation technique 114
2 OLS Results 115
3 IV 117
4 Decomposition of the current account 123
5 The role of credit constraints 125
6 Simulating Current Accounts 126
7 Robustness checks 127
Appendices 135

5 Dynamic Inefficiency and Capital Taxation 157
1 Model 163
2 Results 166
3 Discussion 172
Appendices 176

Conclusion 181

List of Figures

1 Assessing dynamic efficiency in Australia, Canada, Japan, South Korea . . . 16
2 Reproducing and updating Abel et al. (1989) . . . 26
3 Removing land rents at 5% of GDP . . . 27
4 Using OECD data for land rents . . . 27
5 Using OECD data for mixed income . . . 28
6 Preferred specification, assuming Tobin $q = 1$ . . . 28
7 Land as a % of total assets . . . 29
8 Modifying Abel and al. (1989), step by step . . . 30
9 Plotting results from Table 3 . . . 30
10 Reproducing for 6 OECD economies . . . 31
11 Tobin's q, using equity values. . . . 31
12 Fraction of years (in %) in which Investment exceeds Capital Income . . . 32
13 Assessing dynamic efficiency in France, Germany, Hungary and Italy . . . 33
14 Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland . . . 33
15 Assessing dynamic efficiency in Belgium, Denmark, United Kingdom 34
16 Dynamic efficiency as a function of Tobin Q in the US . . . 35
17 Dynamic efficiency as a function of Tobin Q in Japan, Australia and Canada . . . 36
18 Assessing dynamic efficiency in France, Germany, Hungary and Italy . . . 37
19 Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland . . . 37
20 Assessing dynamic efficiency in Belgium, and United Kingdom . . 38
21 Assessing dynamic efficiency in Japan, South Korea, Australia, Canada . . . 38
22 Assessing dynamic efficiency in France, Germany, Hungary and Italy . . . 39
23 Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland . . . 39
24 Assessing dynamic efficiency in Belgium, Denmark, United Kingdom 40

1 Balance sheets: leverage, no short-sales 48
2 Balance sheets: no leverage, short-sales 50
3 Buildup with leverage 58
4 Relaxation of short-sales constraints 60
5 Countercyclical margins 61
6 Lending shut-down 62
7 Effectiveness of margin requirements 79
8 Intuition: leverage and no leverage 80

1 US Business Cycles Facts 88
2 Impulse response functions from a "bubble shock": high interest rates 94
3 Impulse response functions from a "bubble shock": low interest rates 95
4 Jobless recovery 96
5 Impulse response functions from a "productivity shock": low interest rates 103
6 Nominal Interest Rates 105
7 Real Interest Rates 106
8 Housing and Expectations 107
9 Stock Market and Expectations 108
10 Real GDP and Real Wages 109
11 Consumption and Investment 110

1 Instrument, house prices and current accounts in Spain 122
2 Property Taxes as a % of total tax (Blue), Total tax as a % of GDP (Black), and Policy shocks in Spain 122
3 Simulated current accounts and actual ones 127
4 Comparability of house prices indexes 155

1 Capital Taxation in Fisher (1930) Theory of Interest 165
2 OLG model - Dynamic Inefficiency 169
3 OLG model - Dynamic Efficiency 170
4 OLG model - With land 171
5 Treasury Inflation Protected Securities 10-Year 177
6 Property taxes around the world 1/2 178
7 Property taxes around the world 2/2 179

List of Tables

1 House prices and Current Accounts. OLS Regressions. 116
2 Instrumental Variable Approach 120
3 Decomposition of the current account 124
4 Consumer and firm credit constraints 125
5 Granger causality 135
6 Granger causality (Cont.) 136
7 House Prices, Unemployment and Public Saving 136
8 Falsification tests I 137
9 Falsification tests II 137
10 Examining exclusion restriction 137
11 Controlling by different measures of GDP 138
12 Share prices and current accounts? 139
13 Other HP filters 139
14 Other scaling variables: tax as a % of GDP, as a % of Investment and as a % of consumption 140
15 Country Groupings 140
16 Consumer and firm credit constraints: more specifications 141
17 Decomposition of the current account with Year Fixed Effects 142
18 OLS Regressions with controls and year fixed effects 143
19 Instrumental strategy with controls and year fixed effects, First Stage 144
20 Instrumental strategy with controls and year fixed effects, Second Stage 145
21 Instrumental strategy with controls, Investment, Second Stage 146
22 Instrumental strategy with controls, Saving, Second Stage . . . 147
23 Controlling with the Frequency of Cadastral Revisions 148
24 Decades 148
25 Real Exchange Rates 149
26 Recurrent Taxes on Property: Revision of Cadastral Values . . 150
27 Data Appendix: House Price Series 151
28 Test Statistics: Panel of series 153
29 Test Statistics: Individual Tests for Residuals 154

Introduction

There is that scattereth, and yet increaseth;
and there is that withholdeth more than is meet, but it tendeth to poverty.
Proverbs 11:24

There is one very important way in which a market economy can be inefficient even without the "usual suspects" like externalities, information asymmetries, or non-clearing markets: it can fail at allocating resources efficiently across time, for lack of providing agents with a sufficient quantity of stores of value ("asset supply"). The dynamic inefficiency problem can then open up the possibility for indeterminacy of asset prices. The study of this indeterminacy is the topic of this thesis.

Bubbles do not arise in economies considered by Kenneth Arrow and Gérard Debreu (Arrow and Debreu (1954)) because agents are implicitly assumed to be connected by intergenerational links characterized by uniform impatience, which makes the effective number of agents constituting this economy finite.[1] The first and second welfare theorems of economics then hold. In contrast, in overlapping-generations models, in which agents can be connected through intergenerational links different from Barro (1974)-type altruism (like a warm-glow of giving bequests, for example), succeeding generations are different economic entities ; there is a double infinity of goods and agents, so that the proofs of the welfare theorems fail (Shell (1971)).

In Chapter 1, I show that Diamond (1965)'s claim that economies can reach a dynamically inefficient state of capital accumulation is not only theoretical, but that it is likely to be the case for most advanced economies.[2] The literature has mostly focused on the normative implications of such a situation: because the economy has reached a dynamically inefficient state, then issuance of public debt or generalization of pay-as-you-go systems should be Pareto-improving. But I suspect that these direct costs of dynamic inefficiency are likely to be small,

[1] That at least some agents are linked by intergenerational links is unquestionable, however that they are linked through Barro (1974)-type altruism is a stronger assumption. For example, Barro (1974) type agents are uniformly impatient, which would lead them to borrow against their children's income with lower than growth interest rates. This is somewhat counterfactual as some parents do leave some bequests while $r < g$, and forbidden in our legal systems.

[2] Commentators rarely talk about "dynamic inefficiency", but the notions of "abundant liquidity", "reaching for yield" or "chasing returns" all point to this same phenomenon. Note that this is also consistent with ample anecdotal evidence suggesting that the "balance of power" in many ways has shifted to borrowers. Subprime borrowers were able to borrow with no credentials before the US 2007 crisis. Companies are able to issue "cov-lite" loans, that is loans which don't limit the amount of extra-borrowing firms can take, potentially making initial debt a lot more risky. Investors demand bonds that pay in-kind, suggesting that the emphasis is on transferring resources to the farthest possible time. Casual empiricism also suggests that it is not so much capital which is deficient today, but ideas and talents : in today's world of abundant capital, it is not so hard to get financing for a good idea.

and that the reduction in steady-state consumption coming from this phenomenon is not first-order. Moreover, I show for instance in Chapter 2 that there is then a strong force that pushes agents to buy overvalued assets when there is dynamic inefficiency, and that this *de facto* implements the intergenerational transfers that were advocated by Peter Diamond in such a situation. The positive implications have been more overlooked, and these consequences can lead to indirect costs that I would suspect far exceed the first ones. In the rest of the thesis, I aim at showing that the "excess volatility" brought about by dynamic inefficiency can help explain the instability of the financial system (Chapter 2), of financial variables, but also of the real sector. In a closed economy, it can provide a driving force for business cycle fluctuations as sketched in Chapter 3, and for dramatic movements in current accounts as shown in 4. These financial bubbles, leading to back-and-forth movements between tradable and non-tradable industries, are a cause for concern for policymakers and have sometimes been used as a rationale for capital controls.

Though it may not be obvious at first glance, the aim of Chapter 1 is more to communicate a positive message than a normative one.[3] I show in Chapter 2 that rational bubbles are much more stable than one could have thought : far from the very unstable unique path that perfect foresight would lead to bubble to take, there are many paths leading to asset overvaluation when one allows for adaptive learning as a selection criterion among rational expectations equilibria. This means that if old generations' consumption is not increased through more issuance of public debt, the market can and will by itself coordinate on asset prices such that those transfers are implemented. For example, if a rational bubble forms on an asset (like real estate in France to this year 2013), then young agents are de facto transferring resources to their parents when they buy overvalued assets. This equivalence is sometimes loosely referred to when commentators note that not only public indebtedness needs to be reduced, but also private debt. Interestingly, when rational bubbles interacts with deposit insurance, the equivalence between asset overvaluation and public indebtedness (with respect to intergenerational transfers) is even more apparent. Spain and Ireland had contained their indebtedness before the 2007 crisis : for example, according to the OECD, Ireland decreased its public debt from owing a little more than 40% of GDP in 2000 to having a bit more than 30% in 2007. This debt rose to 120% of GDP in 2012 because of the bailing out of the financial sector. The same thing happened in Spain, where debt was decreasing from 2000 at about 60% of GDP to about 40% of GDP in 2007, rising to 95% in 2012. In that case, what the state effectively did was to guarantee the value of the bubble at par, thereby validating the transfer of wealth from the young to the old.

Before proposing "bubbles" as a major candidate for business cycles fluctuations (see Chapter 3), I try to understand more how bubbles can form, and crash, and how agents coordinate on them. In other words, the objective is to make "bubble shocks" more than

[3] On the normative front, I would be very cautious before concluding on whether advanced economies should take on more public debt or not, given the importance of this issue in the context of an aging population. However, I hope my thesis conveys very forcefully that public debt is, in many ways, a rational bubble, and that it is not necessarily a bad thing that it is continuously refinanced. Hence there is in my view no reason to treat this asset as other assets, with the tools of fundamental analysis. The most prominent asset in the world is in fact very hard to understand with the tools of standard economic analysis: most states usually treat their creditors with deference (even when they are non-residents), though public debt is a somewhat unenforceable obligation.

an exercise in residual naming. The model developed in that chapter allows to speak to the effect of financial liberalization, for example the removal of Regulation Q, or to the effects of financial innovation (the removal of short-sales constraints on housing with the creation of the Credit Default Swap on subprime mortgages, in 2006), which have been shown to be important empirical determinants of bubbly episodes (for a survey, see for example Brunnermeier and Oehmke (2013)). Another question this work asks is that of the precise role of the financial system. If we leave really in a world of multiple equilibria, then a lot of risk may be unwarranted and requires costly hedging. This is not only true for financial variables, but also for the real sector. For example, many politicians have argued against flexible exchange rates on this ground precisely: it is very hard to know for an entrepreneur what the exchange rate will be when he starts selling, hence he must purchase costly insurance (Chapter 4 would suggest a similar line of argument).

It is very important to note that the no-bequests assumption that is present in basic OLG models (which is the core of my thesis) isn't at all necessary for the results. All that I need is to deviate from the infinite lives or the consumption-maximizing paradigm in one or in another way. For example, the agents in the models presented could very well care about their children. They could for example have "warm glow" of leaving bequests to them, which would amount to assume that they care more about their children than Barro (1974)-altruistic dynasties, in the current environment with low interest rates.[4] Another way to get the same results, that is a role for asset supply in driving asset prices and business cycles, would be to assume that some agents actually have a utility for wealth per-se, certainly due to the social prestige it helps acquire or because they view it as some kind of certificate of success. The question about the optimality of public debt and rational bubbles in such an environment would then be to know whether these agents care about having real dividends attached to the assets they own or whether the sheer value of their assets suffices to enter in utility. In the second proposition, becoming a landlord in Paris could for example become so pricey (compared to renting) that it would not be worth the discounted stream of future rents, but it would nevertheless give social prestige to its owner. Once again, as I have repeatedly stressed in this introduction, the normative conclusions may be different in that case, but the positive ones will not (for the most part) : in this world, there would still be a strong incentive for agents to acquire overpriced assets, and a role for bubble shocks to drive the business cycle.

Finally, a lot of the policy implications drawn from the analysis contained in this thesis arguably do have some "Keynesian flavor", which I will mention *en passant* as a way to end this introduction. Thus the double infinity of goods and agents will prove to provide an alternative to the sticky prices (or wages) assumptions of Keynesian economics.[5] As a first step, it may

[4]Indeed, Barro-type dynasties would ideally want to borrow very large amount from their children: in a low interest rate environment (lower than the rate of growth), their children would be much richer than they are themselves now. Notwithstanding the legal impossibility of such a proposition (adults cannot leave negative bequests), this remark shows how rational in their love Barro assumes parents are: they care about them, but not so much as to leave them positive bequests when interest rates are low. The presence of both operative bequests and low interest rates in the world should therefore lead one to conclude that at least some agents do not have Barro (1974)-type utility.

[5]It might be all the more important that recent research (for example, Farhi et al. (2011) and Correia et al. (2013)) has suggested that the sticky prices obstacles, and bounds on nominal interest rates could very well be circumvented at zero cost by distortionary fiscal policy.

therefore prove useful to try and understand why John Maynard Keynes did not consider a theory of under-consumption as an alternative to price stickiness. In fact, Keynes (1936) contemplated these views in *The General Theory of Employment, Interest and Money*, but clearly dismissed them. As he explained, theories of oversavings and underconsumption had long been part of the spectrum of economic theories.[6] [7] Complaints of under-consumption constituted some aspect of mercantilist thought. For example, Barthélémy de Laffémas (a counselor to Henri IV, then king of France), in his 1598 opus entitled "Les Trésors et richesses pour mettre l'Estat en Splendeur et monstrer au vray la ruine des François par le trafic et négoce des estrangers" defended that purchasers of French luxury goods helped create a livelihood for the poor. Similarly, Bernard Mandeville's *Fable of the Bees* famously called for living more plentifully; but it was convicted as a nuisance by the grand jury of Middlesex in 1723, which according to John Maynard Keynes "stands out in the history of the moral sciences for its scandalous reputation". Adam Smith famously defended a more austere doctrine in *The Wealth of Nations*:

> Whatever, therefore, we may imagine the real wealth and revenue of a country to consist in, whether in the value of the annual produce of its land and labour, as plain reason seems to dictate; or in the quantity of the precious metals which circulate within it, as vulgar prejudices suppose; in either view of the matter, every prodigal appears to be a public enemy, and every frugal man a public benefactor.

In contrast, Thomas Malthus wrote:

> Adam Smith has stated that capitals are increased by parsimony, that every frugal man is a public benefactor, and that the increase of wealth depends upon the balance of produce above consumption. That these propositions are true to a great extent is perfectly unquestionable... But it is quite obvious that they are not true to an indefinite extent, and that the principles of saving, pushed to excess, would destroy the motive to production. If every person were satisfied with the simplest food, the poorest clothing, and the meanest houses, it is certain that no other sort of food, clothing, and lodging would be in existence... The two extremes are obvious; and it follows that there must be some intermediate point, though the resources of political economy may not be able to ascertain it, where, taking into consideration both the power to produce and the will to consume, the encouragement to the increase of wealth is the greatest.

Finally, it is John Atkinson Hobson who gave more flesh to this argument of over-savings in *The Physiology of Industry*. About his theses, Keynes (1936) writes in *The General Theory of Employment, Interest and Money* :

> Hobson and Mummery were aware that interest was nothing whatever except payment for the use of money. They also knew well enough that their opponents

[6] For some authors, the usefulness of thrift hinges on philosophical or moral considerations, which may make an economic discussion of this matter perhaps a bit partial, to say the least.

[7] The following quotations very much follow Keynes end of Chapter 23 in *The General Theory of Employment, Interest and Money*. For details, I refer the reader to this Chapter.

> would claim that there would be 'such a fall in the rate of interest (or profit) as will act as a check upon Saving, and restore the proper relation between production and consumption'. They point out in reply that 'if a fall of Profit is to induce people to save less, it must operate in one of two ways, either by inducing them to spend more or by inducing them to produce less'. As regards the former they argue that when profits fall the aggregate income of the community is reduced, and 'we cannot suppose that when the average rate of incomes is falling, individuals will be induced to increase their rate of consumption by the fact that the premium upon thrift is correspondingly diminished'; whilst as for the second alternative, 'it is so far from being our intention to deny that a fall of profit, due to over-supply, will check production, that the admission of the operation of this check forms the very centre of our argument'. Nevertheless, their theory failed of completeness, essentially on account of their having no independent theory of the rate of interest; with the result that Mr Hobson laid too much emphasis (especially in his later books) on under-consumption leading to over-investment, in the sense of unprofitable investment, instead of explaining that a relatively weak propensity to consume helps to cause unemployment by requiring and not receiving the accompaniment of a compensating volume of new investment, which, even if it may sometimes occur temporarily through errors of optimism, is in general prevented from happening at all by the prospective profit falling below the standard set by the rate of interest.

I think it can be inferred from this passage that Keynes (1936) rejected the theory of under-consumption as having no independent theory for why the interest rate did not fall enough to restore an equilibrium between savings and investment. John Maynard Keynes died in 1946, and it seems that only with the Allais (1947) - Samuelson (1958) theory of capital was it possible to provide a rigorous framework in which a competitive equilibrium would lead to such a situation of over-saving; and the welfare criterion would favor a situation with lower capital and higher interest rates. Peter Diamond (Diamond (1965)) showed very convincingly how it could be the outcome of a perfectly neoclassical setup with optimizing agents, and competitive and clearing markets.[8]

Even though their rationale can be quite different, it is really striking that policy implications drawn from the dynamic inefficiency analysis are in many ways similar to those proposed by Keynes (1936) as a result of non-clearing markets.[9] [10] When a rational bubble falls, there is similarly a need to use fiscal policy to increase public debt and fill the need for stores of value - which is all the more effective that it occurs through tax decreases, especially on the

[8]Of course, most modern macroeconomists today endorse the Fisher (1930) view of interest rate determination, in which impatience provides a check to the quantity of savings. Allais (1947) to the contrary, thought that scarcity of capital was no permanent feature of our world. For dynamic inefficiency to happen he however added an important qualification: that land should be nationalized. This is discussed at length in Chapter 5.

[9]That overlapping generations model of general equilibrium very much resemble Keynesian economies, is not new: the work of Geanakoplos (2008) made this analogy very clear. As he emphasized, OLG models of general equilibrium leave room for indeterminacy and therefore a role for "beauty contests". The ideas developed in this thesis around OLG models seem to confirm this intuition.

[10]An important caveat is that this view of the world would lead one to be more skeptical view of investment-inducing policies. However, investment *will* naturally rise as a result of the previous policies, because investors will expect agents to consume more in the future and hence investment to be more profitable.

old. As in Keynesian Economics, there is then a lack of eviction effects due to the fact that investment is already at a sufficient level. There is also a role for monetary policy to influence real outcomes, even in a world where prices are fully flexible, as laxer monetary policy allows agents to borrow more heavily from the central bank and helps raise asset prices.[11] The list goes on, and in particular one could easily find in a Bubbly International Business Cycle model (along the lines of the closed economy version developed in Chapter 3) useful analogies with the Mundell-Fleming model of Keynesian Economics. I leave this to future research.

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[11] Note that in this view, monetary policy does not have an effect through the unit of account as in New-Keynesian models but through the leverage of financial institutions and asset prices.

Chapter 2

Bubbles and Adaptive Learning

Abstract

I study adaptive learning of asset prices when priors initially differ, as in Harrison and Kreps (1978). The availability of leverage and short-sales strongly affects asset prices and beliefs on the learning path : in particular, leverage can increase the amount of "overoptimism" in the economy, and feed bubbles' rise. Short-sales can have an opposite effect. In the long run, optimism survives if the economy is dynamically inefficient. In that case, adaptive learning provides a selection criterion for the location of rational bubbles, and a mechanism by which they can grow faster than the rate of interest in a non-stochastic environment. The model also can explain why bubbles usually deflate over weeks sometimes months rather than burst instantaneously. It generates countercyclical margins and procyclical leverage without the assumption of "scary bad news", return predictability, and equilibrium default.

Keywords: Bubbles, heterogeneous expectations, leverage, short-sales constraints.

JEL classification: E3, E44, G01, G21

Acknowledgements: This is a much revised version of a first draft called "Rational Bubbles and Real Estate" in May 2011, later called "The rise and fall of rational bubbles". I am very grateful to Emmanuel Farhi, Guy Laroque, Philippe Martin and Jonathan Parker for very helpful conversations. I also thank John Campbell, Stéphane Gauthier, Robin Greenwood, Roger Guesnerie, Ben Hebert, David Laibson, Andrei Shleifer, Alp Simsek, Jean Tirole, Jaume Ventura, Luis Viceira and Philippe Weil, and to seminar participants at Harvard University, MIT Sloan, and Sciences-Po for useful comments and suggestions. All errors remain mine. I am very grateful to "Corps des Ponts" for funding this research, and to Harvard University for their hospitality.

Introduction

The literature on Harrison and Kreps (1978) - henceforth HK - bubbles, or Tirole (1985) bubbles is silent as to how bubbles appear, and in what circumstances they can crash. In Harrison and Kreps (1978), agents' beliefs exogenously diverges further from period 0's disagreement in period 1; while in Tirole (1985), rational expectation lead agents' optimism to be a constant as a fraction of the size of the economy. In contrast, this paper studies the dynamics of Harrison and Kreps (1978) bubbles when agents learn from past asset prices in an adaptive way. The distribution of beliefs among agents, together with the institutional environment (the availability of short-sales and leverage in particular), determine asset prices; these asset prices in turn determine how the distribution of beliefs gets updated. Under the assumption of flat priors, the dynamics of asset prices under this learning mechanism can be solved for in closed form; while the model can be solved for numerically for other initial prior distributions.

More precisely, I develop a simple asset pricing model where agents have different expectations about the resale value of an asset. Unlike in Geanakoplos (1997) or Simsek (2012), I do not assume that agents face any uncertainty about future payoffs of the asset - in those papers, agents disagree about the probability of a default state - but that they are certain of assets' future payoffs. This, I believe, allows to focus on the most important element of the HK model, namely that agents disagree about mean payoffs.[1] For the sake of generality, I consider disagreement about the resale value of the asset, without specifying whether it is future dividends ("fundamentals") that agents disagree on or the resale value of the asset (possibly including a rational bubble term).[2] The model will allow to study expectations' formation both in the context of the Harrison and Kreps (1978) model as well as in the Tirole (1985) model of rational bubbles. First, in the context of the HK model, in which priors initially differ, for some unmodelled reason, such as overoptimism or other behavioral biases[3], adaptive expectations provide a way to endogenize agents' disagreement, and in particular their "beliefs shocks" in period 1. Since agents' expectations are initially non-rational, adaptive expectations are an interesting candidate for how agents' beliefs are updated in this context.[4]. A second interesting environment to which the model can be applied is one of dynamically inefficient economies with a multiplicity of rational expectations equilibria. In this environment, adaptive learning supplements rational expectations and helps select among many rational expectations equilibria. This selection criterion is discussed at length in Evans and Honkapohja (2001).[5] In this case, a take from the model is that the whole history of leverage and short-sales constraints matters to determine the location of rational bubbles. [6] Moreover, the model with adaptive learning can generate gradual deflation of rational bubbles, which is more in line with the data than the instantaneous fall of stochastic Blanchard and Watson (1983) bubbles.[7]

[1]Agents could very well have some random shocks around this mean, on which they do not disagree. If they are risk neutral, this would not change any of the results from the paper.

[2]Indeed, many popular books about the stock market tend to target a value for a well known indice without any reference to fundamentals (e.g. *Dow 36,000: The New Strategy for Profiting from the Coming Rise in the Stock Market*). I am grateful to John Campbell for this analogy.

[3]Differences in priors can arguably also proxy for differences in initial endowments, risk aversions, which would also lead to trade. However, note that when this initial trade has taken place, there is no further exchange if utility functions are unchanging.

[4]As discussed at length in Bray and Kreps (1989), rational learning to the contrary assumes very high computing power from the part of agents. It would seem counter-intuitive that they would have such very high computing power when learning, that they do not have when calculating the rational expectations equilibrium of the model.

[5]A careful reader might take from reading Evans and Honkapohja (2001) that rational bubbles are neither weakly nor strongly E-stable, however the result relies on the interest rate being non-negative, which rules out dynamic inefficiency. More details are given in Evans (1989).

[6]As discussed in Bray and Kreps (1989), selecting among multiple rational expectations equilibria is something that rational learning also fails to achieve.

[7]As Brunnermeier (2008) notes, this is not only a shortcoming of rational bubbles models, but of all of

The adaptive learning approach provides interesting insights which are hard to explain with other models of asset price formation. First, leverage helps feed overoptimism, and margins are endogenously countercyclical in the model. This is achieved without an assumption of "scary bad news", that is a shock both decreasing the mean and increasing the volatility of expected returns, as in Geanakoplos (1997). Second, the model endogenously features equilibrium default, as default is the certain outcome when a lender provided too much credit to a borrower because he was too optimistic about the future value of the asset. Moreover, even though the loan was thought as fully secured by the lender *ex-ante* (through the collateral's expected resale value), the lender can very well end up making losses *ex-post*, when it turns out he was too optimistic about the future value of the asset. Third, unlike Geanakoplos (1997) and Simsek (2012), I do not need to restrict the contracting environment to simple debt or short-sales contracts: a full set of securities spans the full set of future contingencies, which in this safe environment is assumed to be certain. Alternatively, in a more realistic environment where there would be some risk around the mean, but where this would be an aggregate shock, and agents would be risk neutral[8], agents would also restrict themselves to the simple contracting environment I shall consider here. In other words, I am implicity working here in a complete markets environment.[9] Fourth, the model generates endogenous persistence in asset prices, in particular following some changes in the institutional environment. Let me take the example of a financial liberalization shock, like an decrease in regulatory capital requirements for banks, or the arrival of unregulated hedge funds. Following such a shock, asset prices rise sharply because optimists' (hedge funds) are more able to bet on asset prices using moderates' money (money market funds). But asset prices continue to rise after the shock, because money market funds are in turn less and less worried about the health of hedge funds' assets, as they also update their beliefs in an adaptive way. Therefore, they agree to lend more and more money to hedge funds, as they become more optimistic themselves.[10] Fifth, allowing short-sales has an ambiguous effect on asset prices, as it obviously allows pessimists to express their pessimism about the asset, but it also allows a higher fraction of optimists to express their positive views about asset prices. Depending on their relative magnitudes, and in particular on how leverage is available for short-sellers and agents who are long, short-sales can decrease asset prices or raise them. Finally, the model allows to study the effect of margin requirements on asset prices. This is important since a very debated policy question is whether and if yes how public authorities can help manage financial bubbles. An interesting take from the model is that margin requirements help decrease asset prices when

them. Only a sequence of negative shocks can explain gradual deflation in these models. But they cannot explain protracted busts, like the gradual fall of house prices from 2005 to 2011 in the United States.

[8]This assumption is needed to preserve tractability. Otherwise, the size of agents' positions would influence their risk-taking behavior. This would add more complexity without, I believe, adding more intuition.

[9]This is perhaps some abuse of language, as agents still differ on their assessments of future prices, which is the reason why they do not want to take insurance. Another way to see this in the rational bubbles model is that with multiplicity of rational expectations equilibria, there is no way of insuring against the economy going from one equilibrium to another. In other words, the correspondance mapping the set of parameters describing the economy and prices is not a function. One way to get around this issue is to assume a sunspot process, a measurable exogenous process coordinating agents on one or another equilibrium; but this paper is precisely about defining more precisely how coordination obtains.

[10]This example helps get an intuition of why margins are countercyclical in the model. It is also what happened before the crisis as hedge funds were financing their long term assets making repurchase aggrements with money market funds.

the institutional environment is such that short-sales are not possible; but that they can on the contrary feed bubbles' rise when short-sales are allowed, because they also "throw sand in the wheels" of short-sellers. Therefore, margin requirements may be more effective in cooling down real estate markets (by lowering loan-to-value ratios for example) than in managing stock market exuberance.

Methodologically, the paper adds to HK in many important ways. First, instead of assuming unmodelled short-sales constraints, short-sales constraints here arise endogenously because of the need for pessimists to use collateral to borrow, as in Simsek (2012). Second, I work with a continuum of types (which is necessary because adaptive learning forces to "convexify" the set of beliefs), as in Geanakoplos (1997), who rules out short-sales by assumption. My model is thus in the middle ground between Geanakoplos (1997) and Simsek (2012), which is another reason to consider a risk-free environment - or alternatively, a risky environment with risk-neutral agents, where agents only disagree about the mean of future outcomes.

Related literature. This paper touches upon many strands of the literature which I will only selectively survey here. The disagreement/ short-sales constraints literature started with Miller (1977) and Harrison and Kreps (1978). It is in particular successful at providing a story of the Internet bubble and its subsequent crash in 2001. A review of this literature is in Hong and Stein (2003). In particular, Hong and Stein (2007), Hong et al. (2006), Ofek and Richardson (2003) show that short-sales constraints on real-estate were lifted just when the stock market cashed. It also provides a story for the real-estate boom and bust in the United states in Hong and Sraer (2012). But to the best of my knowledge, none of this literature has studied learning in this type of disagreement models.

The paper borrows from the adaptive expectations literature, a textbook treatment of which is given in Evans and Honkapohja (2001). The origins of adaptive expectations can be traced back to Fisher (1930). The study of their convergence dates back to Bray and Savin (1986) and Fourgeaud et al. (1986).

The rest of the paper proceeds as follows. In section 1, I develop a model of asset-based borrowing, along the lines of Geanakoplos (1997). In section 2, I calculate asset prices in different setups, depending on the availability of asset-based borrowing and cash-collateralized short-selling. In section 3, I study the dynamics of asset prices under econometric learning in those different environments, both when financial constraints are constant and following financial liberalization (increase in leverage) or increased financial complexity (allowance of short-sales). In section 4, I study the same dynamics under constant gain learning. I then investigate the effect of policy measures on stabilizing the leverage cycle in Section 5.

1 The model

Demographics, asset structure. There is a continuum of overlapping generations of agents of measure 1 who work when young, supply their labor inelastically and earn wage income $w_t = (1+g_A)^t$ when young in period t.[11] They care only about old-age consumption $U_t(c_{t+1}) =$

[11] Note that the model generalizes to a model with population growth, in which case the results go through defining overall growth by $1 + g = (1 + g_A) * (1 + g_N)$. It does not matter where growth comes from, be it

$\mathbb{E}c_t = c_t$, and therefore need assets to store the entirety of their labor income. The role of the overlapping-generations structure is to abstract from feedback effects coming from the change in asset prices onto agents wealth, but it is not needed otherwise.[12] Agents can use a storage technology giving $G'(K) < G'(0) = 1 + R'$ per unit investment. Or they can invest in a risky asset, available in supply S, with price p_t in period t.

Note that this set-up can accommodate two types of "bubbles" interpretations. In the HK interpretation, bubbles are due to a possible overoptimism of agents, making them price asset prices above fundamentals. In the case of dynamic inefficiency, bubbles can be rational valuation above fundamental value because the transversality condition is not satisfied. In this interpretation, $1 + R' < (1 + g_N)(1 + g_A)$. In the latter case, the setup is intermediary between Samuelson (1958) and Tirole (1985). In the former, prices are overvalued as in HK, Geanakoplos (1997), Geanakoplos (2003) and Simsek (2012), among other heterogeneous beliefs models.

Beliefs. I assume there is disagreement in each period t as to what p_{t+1} will be, with $\mathbb{E}^i p_{t+1} \in [0, \frac{w_{t+1}}{S}]$.[13] As in Geanakoplos (1997) and Simsek (2012), disagreement is on the mean expectation, but not through the relative probability of different events. I assume that agents are all equally uncertain around their mean priors - for example, they think returns are gaussian around their respective mean beliefs. For simplicity and without loss of generality, I assume their beliefs are degenerate. In Simsek (2012) in contrast, there are 3 types and a continuum of states. In Geanakoplos (1997), there is 2 states and a continuum of types. The reason why I depart from these two benchmarks is that to study adaptive learning, it is important that the set of beliefs be convex. Hence if beliefs are non degenerate (there is some disagreement) then the set of beliefs needs to be a continuum, which implies a continuum of agents. However, using then Geanakoplos (1997) two-state model with a high state and a default state does not allow to talk about short-sales, which are assumed to be exogenously impossible in his model.

Source of disagreement. It does not really matter here whether disagreement is assumed, as in HK; or whether it comes endogenously from the multiplicity of Rational Expectations Equilibria (REE) in the rational bubbles interpretation.[14] Agents draw their priors from the distribution function: $\mathbb{E}^i p_{t+1} \sim F_t(.)$. The corresponding density is $f_t(.)$.

Financial constraints. Borrowing is asset-based, and non recourse, as in Geanakoplos (2003) and Simsek (2012). This type of financial arrangements is prevalent in repurchased agreements, but can also describe some types of non-recourse mortgages.[15] I look at different

demographics or technology.

[12]However, when I talk about dynamic inefficiency, that is the rate of interest rate being negative, there must be some reason why general equilibrium features low interest rates, and the overlapping-generations structure is one of them.

[13]Thinking that the price of an asset will be higher than total endowment next period is an extreme form of irrationality.

[14]Most models of rational bubbles assume this problem away, by assuming perfect foresight: agents are able to perfectly predict actions of unborn agents in future periods. However, there is no coordination device, and today's prices are no information about what agents' expectations will be tomorrow.

[15]Note that the non-recourse character of the loan need not be taken literally. All that matters is that the lender is more worried about the quality of the collateral securing the loan than about the creditworthiness of the borrower.

environments, depending on the availability of short-sell and lending.

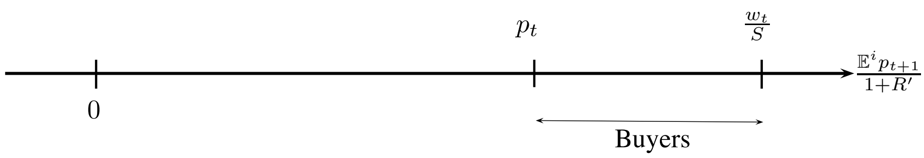
2 Static determination of asset prices

The goal of this section is to determine how agents' expectations of period $t+1$'s price determine period t's price, in different financial environments. I allow for combinations of short-sales, moderate short-sales, and possible leverage. Both short-sales and leverage are limited by the availability of collateral, since all lending is secured. However, I also investigate the case of moderate short-sales, which is meant to capture the fact that short-selling is often only possible on a limited number of units the asset outstanding.

2.1 No lending, no short-sales

Lemma 2.1. *There exist a cutoff expectation for future prices p_t with agents i satisfying $\mathbb{E}^i p_{t+1} \leq (1+R')p_t$ investing in the storage technology ("pessimists"), and agents i satisfying $\mathbb{E}^i p_{t+1} \geq (1+R')p_t$ investing in the bubble asset ("optimists"). This cutoff price satisfies:*

$$\frac{1-F(p_t)}{p_t} = \frac{S}{w_t}. \tag{1}$$



Proof. The condition for the cutoff expectation comes from market clearing for bubbles:

$$\int_{p_t}^{\frac{w_t}{S}} \underbrace{\frac{w_t}{p_t}}_{\#\text{ shares /agent}} \overbrace{f(x)dx}^{\#\text{ agent in }[x;x+dx]} = S.$$

□

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{w_t}{2S}.$$

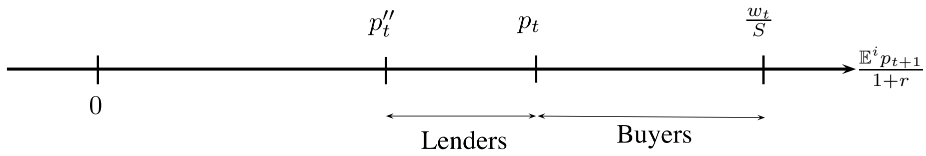
2.2 Lending, no short-sales

Lemma 2.2. *There exist two cutoff expectations for future prices p_t and p_t'' with agents i satisfying $\mathbb{E}^i p_1 \leq (1+R')p_t$ invest in the storage technology ("pessimists"), agents i satisfying $\mathbb{E}^i p_1 \in [(1+R')p_t, (1+R')p_t'']$ lending to optimists ("moderates"), and agents i satisfying*

$\mathbb{E}^i p_1 \geq (1+R')p_t''$ *investing in the bubble asset ("optimists"). These cutoff prices satisfy:*

$$\frac{1-F(p_t'')}{p_t} = \frac{S}{w_t} \tag{2a}$$

$$\int_{p_t''}^{p_t} \frac{f(x)}{x} dx = \frac{S}{w_t} \tag{2b}$$



Proof. The two conditions follow from market clearing for bubbles, and from the collateral constraint. More precisely, the market clearing for value of bubbles reflects the fact that endowments from both lenders and investors end up invested in bubbles:

$$\int_{p_t''}^{\frac{w_t}{S}} \underbrace{w_t}_{\text{endowment /agent}} f(x)dx = \underbrace{p_t S}_{\text{Value of the bubble stock}}.$$

The collateral constraint reflects the scarcity of asset collateral. Only S bubbles can be pledged as collateral for loans[16]:

$$\int_{p_t''}^{p_t} \underbrace{\frac{w_t}{x}}_{\#\text{ bubbles req. in collat.}} f(x)dx = \underbrace{S}_{\text{Total }\#\text{ of bubbles}}.$$

□

The balance sheet of optimists and moderates looks as depicted in Figure 1.

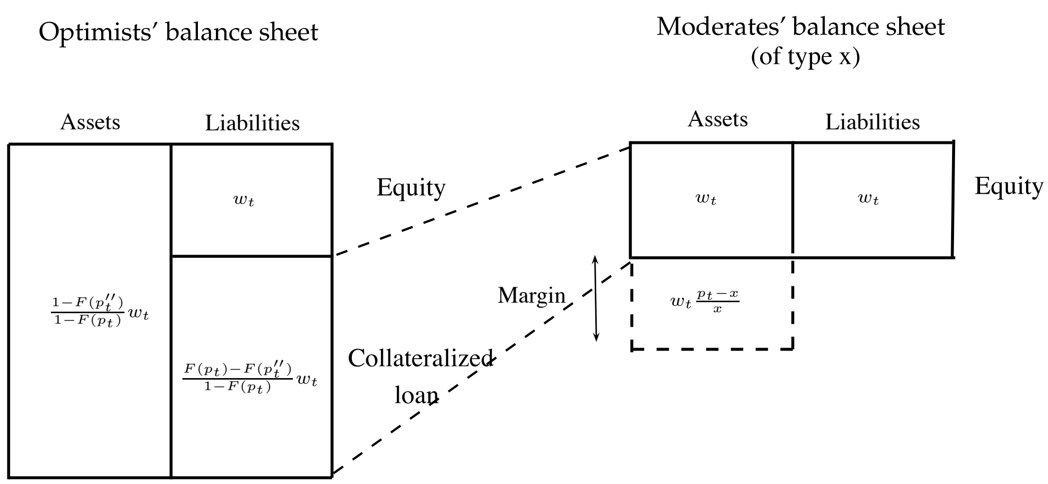
In order to better understand the economic content of these equations, and in particular why leverage allows relatively higher asset prices, it is useful to reason in terms of margins as depicted in Figure 1. Another way to arrive at equation (2a) is then to note that optimists can leverage their investment, investing $w_t \frac{1-F(p_t'')}{1-F(p_t)}$ instead of their endowment w_t. It suffices then to multiply the number of optimists by the number of assets each of them can buy to arrive at the same equation:

$$(1-F(p_t))\frac{w_t \frac{1-F(p_t'')}{1-F(p_t)}}{p_t} = S.$$

Another way to arrive at equation (2b) is to equate the mean per-lender margin required by lenders on asset-backed borrowing and note that in equilibrium, it should be equal to the

[16]In practice, a borrower cannot pledge an asset many times either because it finances this asset through a repurchase agreement where the asset stays with the lender, or because these trades occur through a clearing house.

Figure 1: BALANCE SHEETS: LEVERAGE, NO SHORT-SALES



borrowers' own funds (adjusting for the ratio of the number of borrowers to the number of lenders):

$$\frac{1}{F(p_t)-F(p_t'')}\int_{p_t''}^{p_t} w_t \frac{p_t - x}{x} f(x)dx = w_t \frac{1-F(p_t)}{F(p_t)-F(p_t'')}.$$

When combined with (2a), this equation yields to equation (2b) - see Appendix 2.3 for details.

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{e}{e+1}\frac{w_t}{S} \qquad p_t'' = \frac{1}{e+1}\frac{w_t}{S}$$

Proof. See Appendix 2.1. □

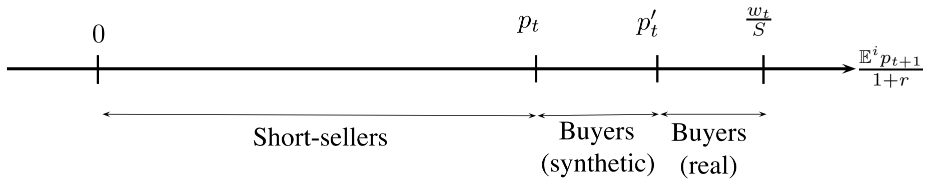
2.3 No lending and short-sales

Lemma 2.3. *There exist two cutoff expectations for future prices* p_t *and* p_t' *with agents* i *satisfying* $\mathbb{E}^i p_1 \leq (1+R')p_t'$ *shorting the asset ("pessimists"), agents* i *satisfying* $\mathbb{E}^i p_1 \in [(1+R')p_t', (1+R')p_t]$ *being long the asset through replicas - equivalently, lending the asset to pessimists - ("moderates") , and agents* i *satisfying* $\mathbb{E}^i p_1 \geq (1+R')p_t$ *investing in the real asset ("optimists"). These cutoff prices satisfy:*

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \tag{3a}$$

$$\int_0^{p_t} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'} (x-p_t)f(x)dx. \tag{3b}$$

Proof. With no-lending and short-sales, pessimists sell-short, and optimists are long in the asset. Agents with expectations higher than p_t' hold the non-synthetic asset. In contrast,



agents between p_t and p_t' hold the synthetic asset. The intuition is that less optimistic agents ask less guarantees to pessimist to lend them the asset, since they think pessimists will make lower losses. Market clearing for bubbles says that optimists' endowments are invested in the bubble asset:

$$\int_{p_t'}^{\frac{w_t}{S}} \underbrace{w_t}_{\text{endowment /agent}} f(x)dx = \underbrace{p_t S}_{\text{Value of the bubble stock}}.$$

Agents holding the synthetic asset, or equivalently lending the real asset to short-sellers (which, in equilibrium, end up in the hands of optimists), are worried that the price of the asset might increase which will force pessimists into default. Therefore, in addition to the proceeds from the sale of the asset by the short-sellers, moderates with beliefs x ask for $x - p_t$ in collateral for each unit of asset lent.[17] Each of them lend w_t/p_t units of the asset, or buy the equivalent amount of replicas ; this collateral equals the endowment of pessimists in equilibrium, which is the maximum collateral that they can pledge (note that the proceeds of the sale is assumed to stay with the lender, but it is irrelevant):

$$\int_{p_t}^{p_t'} \underbrace{\frac{w_t}{p_t}}_{\#\text{ each holds}} (x - p_t) f(x)dx = w_t \int_0^{p_t} f(x)dx.$$

□

In Appendix 1, I treat the case of other forms of short-sales. In particular, the case considered here corresponds to that of stock lending, whereby the borrower need not be worried about lenders' collateral, because the lender lends actual security that he possesses as a consequence. As I show in the appendix, one could as well treat a case where it is in contrast the short-seller who is worried about the quality of the lender providing "replicas"; as Lewis (2010) documents, it was indeed very hard for short-sellers to find a suitable counterparty for short-selling real estate. One can consider a case in which both lenders and borrowers worry about the quality of their counterparty to a synthetic contract. As is intuitive, prices are lower when short-sellers do not worry about lenders' collateral, and higher when lenders do not worry about the creditworthiness of short-sellers.

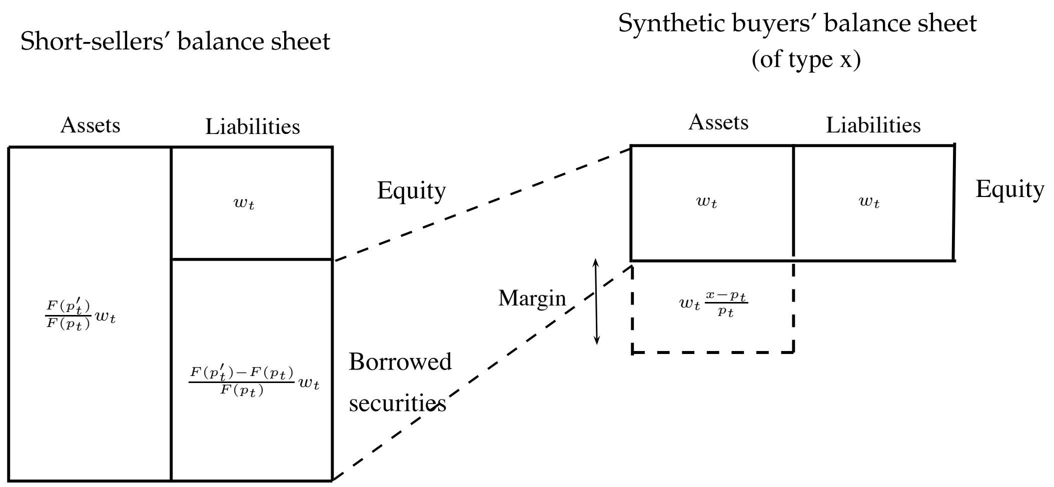
Note that the number of synthetic contracts is given by:

$$S' = \frac{w_t}{p_t}(F(p_t') - F(p_t)).$$

[17] Note that although possible losses from a short-sale are theoretically infinite, here agents are not too optimistic so they will ask for a finite value of collateral.

The balance sheet of short sellers and lenders of securities is depicted in Figure 2.

Figure 2: BALANCE SHEETS: NO LEVERAGE, SHORT-SALES



Once again, in order to better gain the economic meaning for the environment, let me derive the same equilibrium equations using margins. The first equation is straightforward, as it just says that optimists' equity is invested in the real asset. More interestingly, one can arrive at equation (3b) using that mean margins required by synthetic buyers on short-sellers should equal short-sellers own funds (there is an adjustment for the ratio of the number of short-sellers to the number of synthetic buyers):

$$\frac{1}{F(p'_t)-F(p_t)}\int_{p_t}^{p'_t} w_t\frac{x-p_t}{p_t}f(x)dx = w_t\frac{F(p_t)}{F(p'_t)-F(p_t)}.$$

In particular, for the case of flat priors, that is, $f(x)=S/w_t$, one gets that:

$$p_t=\frac{1}{\sqrt{2}\,(1+\sqrt{2})}\frac{w_t}{S} \qquad p'_t=\frac{1}{\sqrt{2}}\frac{w_t}{S} \qquad S'=\sqrt{2}S.$$

Proof. See Appendix 2.1. □

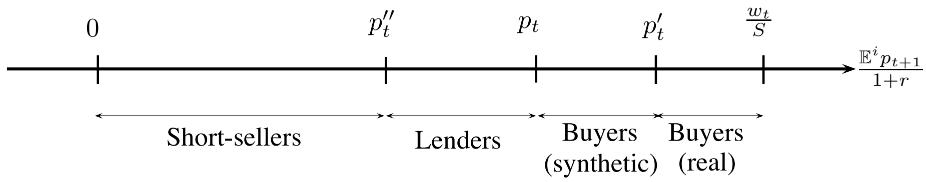
2.4 Lending, short-sales

Lemma 2.4. *With both lending and short-sales, pessimists sell-short, negative moderates lend cash to optimists, positive moderates lend the asset to pessimists, and optimists buy the real* *asset. The cutoffs are such that:*

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \tag{4a}$$

$$\int_0^{p_t''} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'} (x-p_t)f(x)dx \tag{4b}$$

$$\int_{p_t''}^{p_t} \frac{f(x)}{x}dx = \frac{S}{w_t}. \tag{4c}$$



Proof. With lending and short-sales, pessimists sell-short, moderates lend to optimists and optimists are long in the asset. Agents with expectations higher than p_t' hold the non-synthetic asset. In contrast, agents between p_t and p_t' hold the synthetic asset. As in the previous section, the intuition is again that less optimistic agents ask less guarantees to pessimists to lend the asset, since they think pessimists will make lower losses. Again, market clearing for bubbles says that optimists' endowments are invested in the bubble asset as in the previous section, which gives equation (4a). Short-sellers are agents in $[0, p_t'']$ this time, and they must use cash-collateral to protect their loans. Apart from that, the logic is similar, so that equation (4b) holds.

Finally, I assume that lenders can only lend against "real" assets. This assumption is justified by the fact that the cash-collateral stays entirely with the lender (or his broker). This arrangement ensures the short-seller that he will be repaid in full even if the asset drops in value. Therefore:

$$\int_{p_t''}^{p_t} \underbrace{\frac{w_t}{x}}_{\#\text{ bubbles req. in collat.}} f(x)dx = \underbrace{S}_{\text{Total }\#\text{ of bubbles}}.$$

□

Note that the number of synthetic contracts is again given in equilibrium by:

$$S' = \frac{w_t}{p_t}(F(p_t') - F(p_t)).$$

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{e}}+\sqrt{2}\right)}\frac{w_t}{S} \qquad p_t' = \frac{1}{\sqrt{2}}\frac{\sqrt{2}+\sqrt{e}}{1+\sqrt{2e}}\frac{w_t}{S} \qquad p_t'' = \frac{1}{e\sqrt{2}\left(\frac{1}{\sqrt{e}}+\sqrt{2}\right)}\frac{w_t}{S} \qquad S' = S.$$

Proof. See Appendix 2.1. □

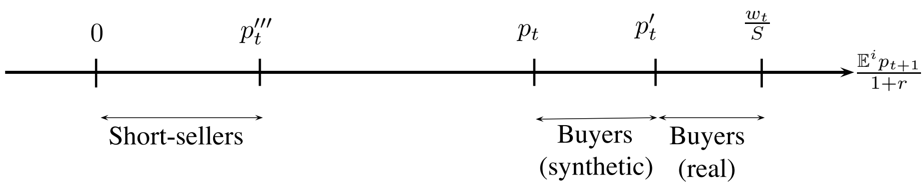
2.5 No lending, exogenous short-sales

Lemma 2.5. *With no lending and limited short-sales, the cutoffs are such that:*

$$\frac{1-F(p'_t)}{p_t} = \frac{S}{w_t} \tag{5a}$$

$$\frac{1-F(p_t)}{p_t} = \frac{S+S'}{w_t} \tag{5b}$$

$$\int_0^{p'''_t} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p'_t}(x-p_t)f(x)dx. \tag{5c}$$



Proof. With no lending and moderate short-sales, pessimists sell-short and optimists are long in the asset. Agents with expectations higher than p'_t hold the non-synthetic asset. In contrast, agents between p_t and p'_t hold the synthetic asset. As in the previous section, the intuition is again that less optimistic agents ask less guarantees to pessimists to lend the asset, since they think pessimists will make lower losses. However, short-selling is limited to an equivalent number S' of real assets.[18] Equilibrium is given by market clearing for real bubbles, or equation (5a) : optimists' endowments are invested in the bubble asset as in the previous section. Market clearing for synthetic and real bubbles gives:

$$\frac{1-F(p_t)}{p_t} = \frac{S+S'}{w_t}.$$

Finally, collateral requirements are, just as in the previous section:

$$\int_0^{p'''_t} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p'_t}(x-p_t)f(x)dx.$$

□

Another identity following from the two first previous relates the number of bought/sold short contracts to how many assets lenders can lend.

$$S' = \frac{w_t}{p_t}(F(p'_t)-F(p_t)).$$

[18]In practice, only institutional investors lend stocks for example, so a small fraction of them is available for short-selling. S' could also be a reduced form representation of the cost of writing derivative contracts.

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{S}{2S+S'}\frac{w_t}{S} \qquad p_t' = \frac{S+S'}{2S+S'}\frac{w_t}{S} \qquad p_t''' = \frac{S'^2}{2S(2S+S')}\frac{w_t}{S}$$

Proof. See Appendix 2.1. □

The condition for limited short-sales constraints in the case of flat priors therefore writes :

$$2S^2 + 2SS' - S'^2 > 0 \quad \Leftrightarrow \quad S' < \frac{2}{\sqrt{3}-1}S.$$

2.6 Lending, exogenous short-sales

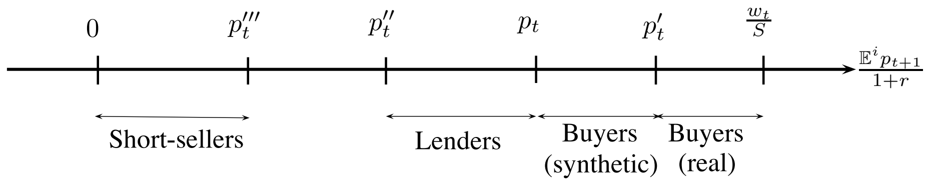
Lemma 2.6. *With lending and limited short-sales, the cutoffs are such that:*

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \tag{6a}$$

$$\frac{1-F(p_t'')}{p_t} = \frac{S+S'}{w_t} \tag{6b}$$

$$\int_0^{p_t'''} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'}(x-p_t)f(x)dx \tag{6c}$$

$$\int_{p_t''}^{p_t} \frac{f(x)}{x}dx = \frac{S}{w_t}. \tag{6d}$$



Proof. With lending and moderate short-sales, pessimists sell-short, moderates lend to optimists and optimists are long in the asset. Once again, agents with expectations higher than p_t' hold the non-synthetic asset. In contrast, agents between p_t and p_t' hold the synthetic asset. As in the previous section, the intuition is again that less optimistic agents ask less guarantees to pessimists to lend the asset, since they think pessimists will make lower losses. However, short-selling is limited to an equivalent number S' of real assets.[19] Equilibrium is given by market clearing for real bubbles in equation (6a) : optimists' endowments are invested in the bubble asset as in the previous section. Market clearing for endowments of lenders, synthetic and real buyers gives:

$$\frac{1-F(p_t'')}{p_t} = \frac{S+S'}{w_t}.$$

[19] In practice, only institutional investors lend stocks for example, so a small fraction of them is available for short-selling. S' could also be a reduced form representation of the cost of writing derivative contracts.

Collateral requirements are the same as in the previous section, which gives equation (6c). Finally, lenders can only lend against real assets and therefore:

$$\int_{p_t''}^{p_t} \frac{f(x)}{x} dx = \frac{S}{w_t}.$$

□

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{e}{1 + \frac{S+S'}{S}e}\frac{w_t}{S} \qquad p_t' = \frac{1 + \frac{S'}{S}e}{1 + \frac{S+S'}{S}e}\frac{w_t}{S} \qquad p_t'' = \frac{1}{1 + \frac{S+S'}{S}e}\frac{w_t}{S}$$

$$p_t''' = \frac{1}{2e}\frac{\left(1 + \frac{S'-S}{S}e\right)^2}{1 + \frac{S+S'}{S}e}\frac{w_t}{S}$$

Proof. See Appendix 2.1. □

The condition for limited short-sales constraints in the case of flat priors therefore writes :

$$\frac{1}{2e}\left(1 + \frac{S' - S}{S}e\right)^2 < 1 \quad \Leftrightarrow \quad S' < \left(\frac{\sqrt{2e} - 1}{e} + 1\right) S.$$

2.7 Discussion

It is possible to get closed-form solutions for the cutoffs when there are flat priors, as I have shown throughout. This is interesting, as flat priors are a reasonable candidate for initial priors in the case of dynamic inefficiency where there is a continuum of REE in $[0, \frac{w_t}{S}]$.

For more general prior distributions, one generally falls on non-linear equations which cannot be solved for in closed form. However, these are very easy to solve on the computer.

3 Econometric learning

In this section, agents learn econometrically, that is they seek to estimate the historical (statistical) mean of prices. As a consequence, their expectations are :[20]

$$p_{t+1}^e = \frac{1}{t}\sum_{j=0}^{t-1} p_j.$$

Note that, as Evans and Honkapohja (2001) remark, this method of learning is of the form:

$$\begin{aligned} p_{t+1}^e &= p_t^e + \gamma_t(p_{t-1} - p_t^e) \\ &= (1 - \gamma_t)p_t^e + \gamma_t p_{t-1} \end{aligned}$$

[20]Note that the timing is such that agents do not use today's prices to forecast future prices. Adding such a refinement would lead to a more complicated fixed point problem but would not bring additional insights.

Denoting $\gamma_t = 1/t$ the gains of the learning process.

For simplicity in notation, assume that $w_t = 1$, that is there is no growth. The results easily generalize to an environment with growth.

Lemma 3.1. *This econometric learning rule defines a mapping from period t's distribution function of beliefs to the next:*

$$F_t(x) = \begin{cases} 0 & \textit{if} \quad 0 \leq x \leq \gamma_t p_{t-1} \\ F_{t-1}\left(\frac{x-\gamma_t p_{t-1}}{1-\gamma_t}\right) & \textit{if} \quad \gamma_t p_{t-1} \leq x \leq \gamma_t p_{t-1} + 1 - \gamma_t \\ 1 & \textit{if} \quad x \in \gamma_t p_{t-1} + (1-\gamma_t) \leq x \leq 1 \end{cases}$$

A corollary is that there is also a mapping between density function:

$$f_t(x) = \begin{cases} 0 & \text{if} \quad 0 \leq x \leq \gamma_t p_{t-1} \\ \frac{1}{1-\gamma_t} f_{t-1}\left(\frac{x-\gamma_t p_{t-1}}{1-\gamma_t}\right) & \text{if} \quad \gamma_t p_{t-1} \leq x \leq \gamma_t p_{t-1} + 1 - \gamma_t \\ 0 & \text{if} \quad x \in \gamma_t p_{t-1} + 1 - \gamma_t \leq x \leq 1 \end{cases}$$

Proof. See Appendix 2.3 □

Some results can be obtained with general prior distributions, while some others require more assumptions. Once again, the case of flat priors will prove interesting. Note that:

$$\forall x \in [0,1], \quad f_t(x) = \frac{1}{1-\gamma_t} f_{t-1}\left(\frac{x-\gamma_t p_{t-1}}{1-\gamma_t}\right) \mathbb{1}[\gamma_t p_{t-1}, \gamma_t p_{t-1} + 1 - \gamma_t](x).$$

Therefore, starting from a flat prior distribution over $[0,1]$, the distribution of beliefs in period t will be:

$$\begin{aligned} f_t(x) &= \frac{1}{a_t}\mathbb{1}[b_t, a_t + b_t](x) \\ \text{with} \quad a_{t+1} &= \frac{1}{1-\gamma_t} a_t, \quad a_1 = 1 \\ b_{t+1} &= b_t + \gamma_t p_{t-1}, \quad b_1 = 0. \end{aligned} \tag{7}$$

The corresponding distribution function is:

$$F_t(x) = \frac{1}{a_t}(x - b_t)\mathbb{1}[b_t, a_t + b_t](x) + \mathbb{1}[a_t + b_t, 1](x).$$

Note that the sequence of prices $\{p_t\}_{t=1}^{\infty}$ is determined below and differs depending on the level of financial constraints.

3.1 No lending, no-short-sales

With no lending and no short-sales, the price of the asset stays constant through the learning process. Agents eventually come to agree with each other, but there is no effect on asset prices. This means that agents investing in the asset will persistently earn more money than those who are not if the economy is dynamically inefficient, and the reverse if the economy is dynamically efficient. Note that such is not necessarily inconsistent with optimization and arbitrage, but is a result of the learning assumption: agents do not seek to learn about returns, but about the "right" level of prices.

Proposition 4 (No leverage, no short-sales)**.** *With no leverage and no short-sales, the asset price level is constant.*

Proof. By recursion, I assume that p_t is an equilibrium price at date t, I will prove that it is also an equilibrium price at date $t+1$. From 1 and $w_t = S = 1$ by assumption (see the beginning of this section, this is of course not required), and the fact that p_t and p_{t+1} are equilibria at date t and $t+1$ respectively,

$$1 - F_t(p_t) = p_t$$
$$1 - F_{t+1}(p_{t+1}) = p_{t+1}$$

Calculate:

$$F_{t+1}(p_t) = F_t\left(\frac{p_t - \gamma_{t+1}p_t}{1 - \gamma_{t+1}}\right) = F_t(p_t) = 1 - p_t$$

Since $F_{t+1}(p_{t+1}) = 1 - p_{t+1}$, from monotonicity and uniqueness of p_{t+1} follows that $p_{t+1} = p_t$. $\square$

Intuitively, under the adaptive learning rule developed above, the quantile $F(p_t)$ is unchanged even as F is updated. This is because p_t "attracts" the distribution. Of course, this intuition will therefore be true under more general learning rules (see Section 4 for the case of constant gain learning). So it suffices to calculate the price for the initial prior distribution, and this price will remain constant in the following. In the case of flat initial priors for example, the price equals 0.5 in period 1 and ever after.

3.2 Lending, no short-sales

With leverage and no short-sales, the econometric learning dynamics are more complex. Let me first go through a simple and natural example in which agents initially have flat priors about prices.

Proposition 5 (Leverage, no short-sales, flat priors)**.** *With leverage, no short-sales, and flat priors initially, asset prices are increasing over time before reaching a plateau.*

Proof. From equation (2b):

$$\int_{p_t''}^{p_t} \frac{f_t(x)}{x} dx = \frac{S}{w_t},$$

and therefore $p_t'' = e^{-a_t} p_t$. (note that p_t and p_t'' are necessarily in $[b_t, a_t + b_t]$). Replacing in equation (2a) yields:

$$1 - \frac{1}{a_t}(p_t'' - b_t) = p_t \quad \Leftrightarrow \quad p_t = \frac{a_t + b_t}{a_t + e^{-a_t}}.$$

With $\{a_t\}_{t=1}^{\infty}$ and $\{b_t\}_{t=1}^{\infty}$ satisfying the recurrence relations:

$$\begin{cases} a_{t+1} = (1-\gamma_t)a_t \\ b_{t+1} = b_t + \gamma_t \frac{a_{t-1}+b_{t-1}}{a_{t-1}+e^{-a_{t-1}}}. \end{cases}$$

From $\gamma_t = 1/t$, $a_{t+1} = 1/t$ and so from $b_t \leq p_t \leq b_t + a_t$ so if $\{b_t\}_{t=1}^{\infty}$ converges then $\{p_t\}_{t=1}^{\infty}$ has the same limit.

□

In this economy, there are many relevant margins in the market, because optimists borrow from pessimists with different valuations of the collateral. However, for clarity, I will talk about "the margin" in the economy defined as follows:

Definition 3.2 (Margin)**.** *I define the "margin" in the economy as the highest margin of the loan taking place between the most pessimistic of all lenders and an optimist:*

$$m_t = 1 - \frac{p_t''}{p_t}.$$

The economy described here features countercyclical margins (or procyclical loan to value ratios), as agents learn the true distribution in the economy, with initial flat priors:

Proposition 6 (Countercyclical margins with initial flat priors)**.** *With leverage, no short-sales, and flat priors, margins are countercyclical: $m_{t+1} \leq m_t$.*

Proof. The proposition immediately follows from equation (2a) yielding $p_t'' = e^{-a_t}p_t$, and $a_t = 1/(t-1)$, so that the margin can be solved in closed form :

$$m_t = 1 - \exp\left(-\frac{1}{t-1}\right)$$

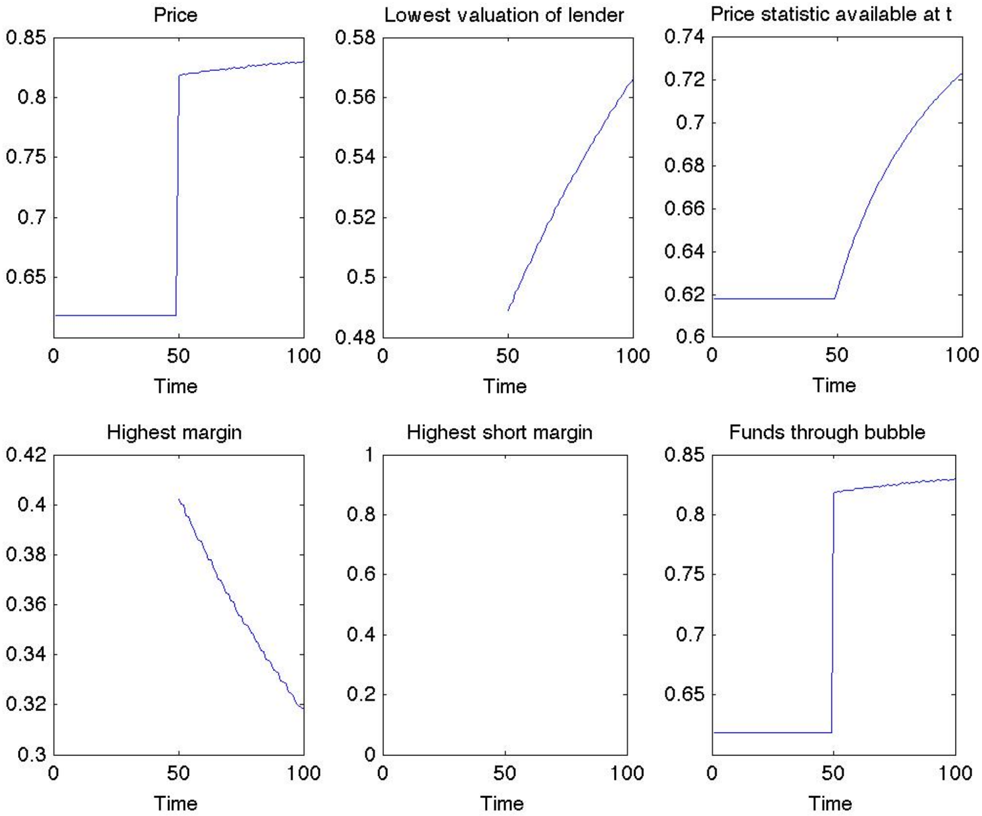
Which is a decreasing function in t. □

The intuition behind this result is that in contrast to the previous case, here the whole distribution matters with leverage or short-sales. With leverage :

$$\frac{1 - F(p_t')}{p_t} = 1 \qquad \int_{p_t'}^{p_t} \frac{f(x)}{x} dx = 1.$$

The rise in the bubble is illustrated on Figure 3. The intuition behind the result is best illustrated in Figure 8: the whole distribution of priors matters for determining prices, not just the quantile.

Figure 3: BUILDUP WITH LEVERAGE



Note: Prior distribution is $\forall x \in [0,1], f(x) = 2x$. Bayesian learning parameters are $\sigma = 1$, $\sigma = 0.1$. Short-sales are never possible. For $t \in [0, 50]$, leverage is not possible. Asset-collateralized leverage is possible from $t \geq 50$ on. Prices increase at impact since optimists buy more assets through leverage, but prices continue their rise after impact.

3.3 No lending, short-sales

Again, with flat priors can one get closed form solutions for recursive equations. Equation (3b) gives:

$$\int_0^{p_t} f_t(x)dx = \frac{1}{p_t}\int_{p_t}^{p'_t}(x - p_t)f_t(x)dx.$$

Which one can write as :

$$\frac{1}{a_t}(p_t - b_t) = \frac{1}{p_t}\frac{(p'_t - p_t)^2}{2}\frac{1}{a_t}$$

Equation (3a) writes: $1 - F_t(p'_t) = p_t$. Therefore:

$$p'_t = b_t + (1 - p_t)a_t$$

Combining those two equations gives a quadratic equation that p_t solves (see Appendix 2.3 for details) :

$$\left[(a_t+1)^2-2\right]p_t^2-2a_t(a_t+b_t+1)p_t+(a_t+b_t)^2=0.$$

The discriminant for this equation is:

$$\Delta=4a_t^2(a_t+b_t+1)^2+4(a_t+b_t)^2[2-(a_t+1)^2].$$

The discriminant is positive for $t\geq 3$ because then $a_t<\sqrt{2}-2$. One can verify very easily that the discriminant is positive for any $t=2$ or $t=3$, since $b_t\leq 1$. (it suffices to calculate the two first values for the sequences $\{a_t\}_{t=2}^{\infty}$ and $\{b_t\}_{t=2}^{\infty}$.

Then one of the two previous equations gives p_t', and the recursions for $\{a_t\}_{t=2}^{\infty}$ and $\{b_t\}_{t=2}^{\infty}$ are solved in closed form from equation (7).

3.4 Lending, short-sales

Again, I compute the dynamics for flat priors in closed form. The non linear system is the following :

$$\begin{aligned}\frac{1}{a_t}(p_t''-b_t)&=\frac{1}{p_t}\frac{(p_t'-p_t)^2}{2}\frac{1}{a_t}\\ p_t''&=e^{-a_t}p_t\\ p_t'&=b_t+(1-p_t)a_t\end{aligned}$$

They lead to a quadratic equation for p_t:

$$\left[(a_t+1)^2-2\right]p_t^2-2a_t(a_t+b_t+1)p_t+(a_t+b_t)^2=0.$$

Very interestingly, the discriminant of this equation is not necessarily positive, and therefore the problem does not always admit an equilibrium:

$$\Delta=4a_t^2(a_t+b_t+1)^2+4(a_t+b_t)^2[2e^{-a_t}-(a_t+1)^2].$$

There is no equilibrium if and only if (see Appendix 2.3 for more detail) :

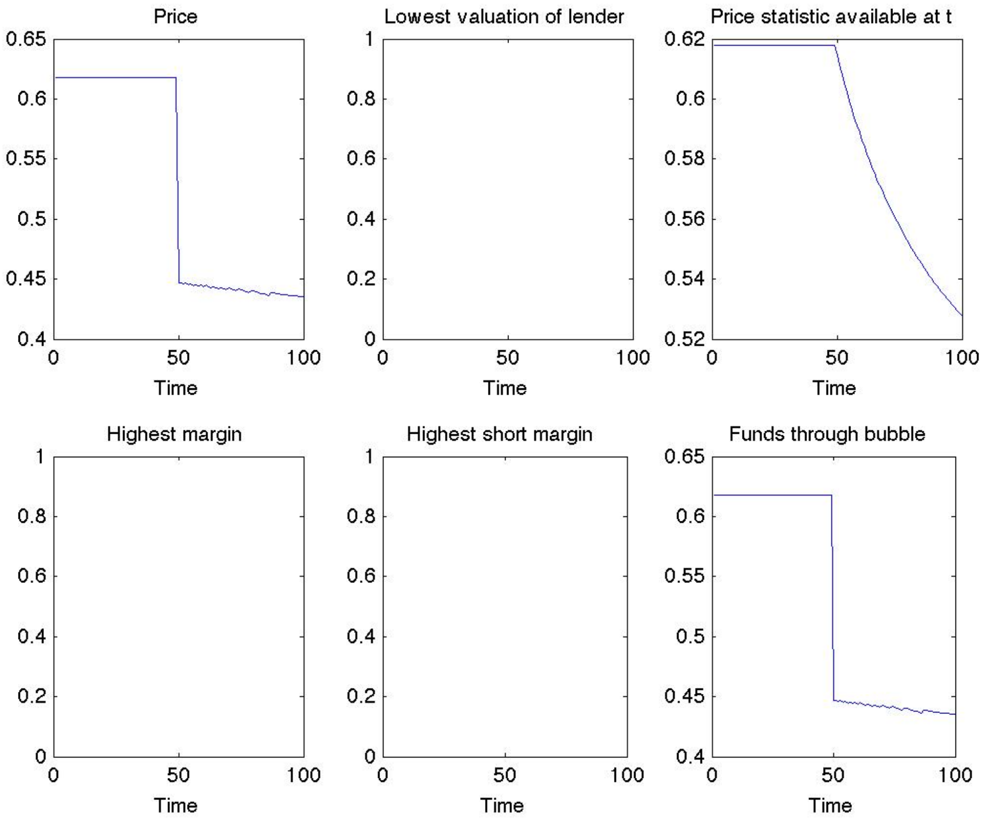
$$b_t\geq\frac{a_t+1+\sqrt{(a_t+1)^2-2e^{-a_t}}}{\sqrt{(a_t+1)^2-2e^{-a_t}}-a_t}\quad\text{and}\quad(a_t+1)^2\leq a_t^2+2e^{-a_t}.$$

Note however that this is specific to the usual case where shorting occurs through borrowing in securities, and lenders therefore must put up p_t for each security lent, while short-sellers would not be so pessimistic if they were to choose the required amount of collateral. In the case where there is symmetric betting on the two sides, as in Appendix 1, equations are linear. (see Appendix **??**)

3.5 Relaxation of short-sales constraints

When short-sales constraints are only limited by available cash-collateral, asset bubbles can burst progressively rather than instantaneously.

Figure 4: Relaxation of short-sales constraints



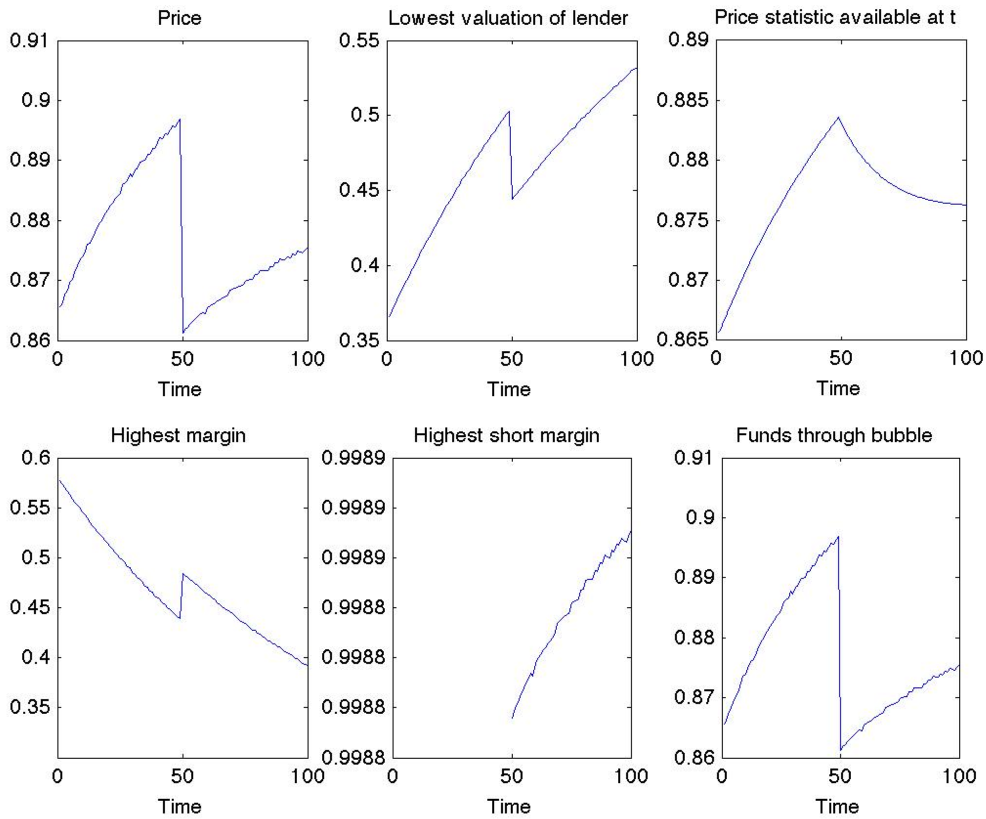
Note: Prior distribution is $\forall x \in [0,1], f(x) = 2x$. Bayesian learning parameters are $\sigma = 1$, $\sigma = 0.1$. Leverage is never possible. For $t \in [0, 50]$, short-sales are not possible. Cash-collateralized short-sales with $S' = 0.1$ (see the main text) are possible from $t \geq 50$ on. Prices decrease gradually.

I follow Fostel and Geanakoplos (2011) in assuming that financial innovation came sequentially: that is, I assume that only leverage is possible for $t < 50$, and that short-selling is possible up to fraction S' of the asset stock from date $t \geq 50$ on.

4 Constant gain learning

Agents can alternatively use constant gain learning (intuitively, they update more quickly) when confronted with possible structural changes, as in some of the sequences of events experimented before (relaxation of short-sales, lending, etc.).This time, learning takes the

Figure 5: Countercyclical margins



Note: Prior distribution is $\forall x \in [0,1], f(x) = 2x$. Bayesian learning parameters are $\sigma = 1$, $\sigma = 0.1$. For $t \in [0, 50]$, leverage is possible but not short sales. Moderate short-sales with $S' = 0.1$ (see the main text) are possible from $t \geq 50$ on, as in Fostel and Geanakoplos (2011). Margins are countercyclical, as can be seen in the bottom-left panel, both on the rise and at impact.

form:

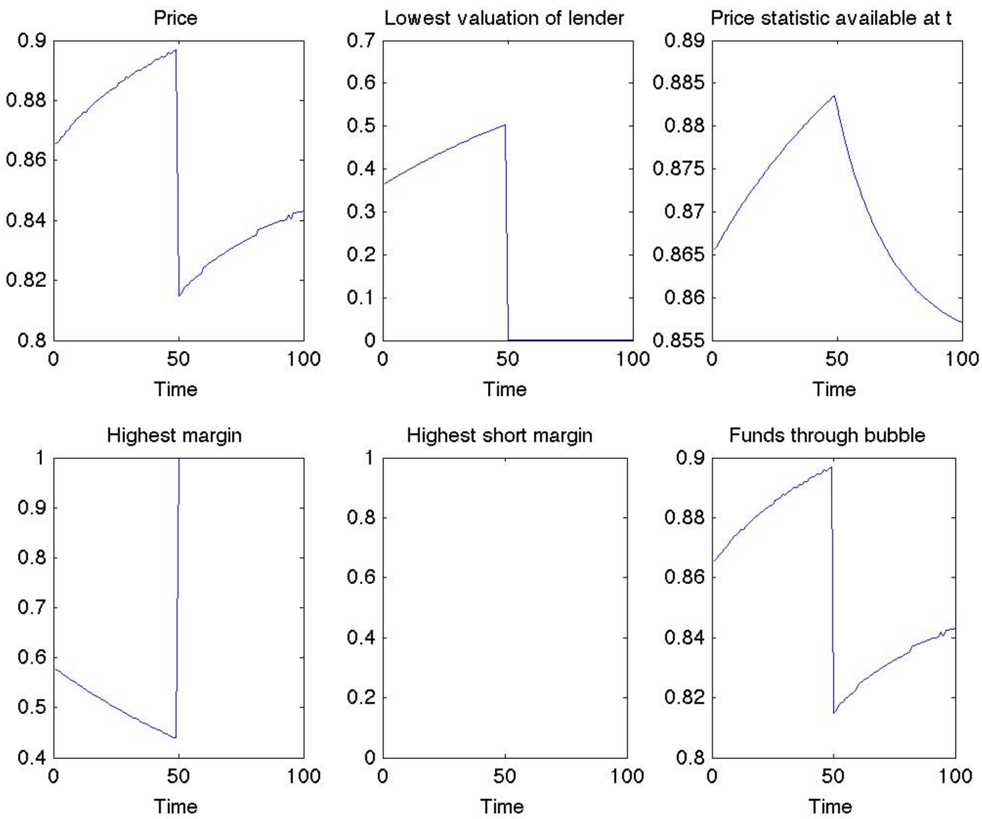
$$\begin{aligned} p^e_{t+1} &= p^e_t + \gamma(p_{t-1} - p^e_t) \\ &= (1-\gamma)p^e_t + \gamma p_{t-1}, \end{aligned}$$

where $\gamma_t = \gamma$ is the constant gain learning parameter. Note that as is intuitive, agents in these models will learn much more quickly, but as a consequence dynamics will have the potential of being much less stable.

4.1 No lending, no short-sales

In the case of no lending and no short-sales, the price remains constant as in the case of econometric learning. As discussed above, the mechanism outlined is much more general than the case of geometrically decreasing gains and relies on the updating preserving the quantile

Figure 6: LENDING SHUT-DOWN



Note: Prior distribution is $\forall x \in [0,1], f(x) = 2x$. Bayesian learning parameters are $\sigma = 1$, $\sigma = 0.1$. For $t \in [0,50]$, leverage is possible but not short sales. Cash-collateralized short-sales constraints (see the main text) are possible from $t \geq 50$ on, as in Fostel and Geanakoplos (2011). Margins are extremely countercylical at impact in the sense that the market for asset-based lending shuts down at $t = 50$.

$F_t(p_t)$. With constant gain learning , we have similarly that

$$F_{t+1}(p_t) = F_t\left(\frac{p_t - \gamma p_t}{1-\gamma}\right) = F_t(p_t) = 1 - p_t$$

Since $F_{t+1}(p_{t+1}) = 1 - p_{t+1}$, from monotonicity and uniqueness of p_{t+1} follows that $p_{t+1} = p_t$.

4.2 Lending, no short-sales

Similarly as in the case of geometrically decreasing gains, one can calculate the path for asset prices from the recursive equations and the form of sequence $\{a_t\}_{t=1}^{\infty}$. Note that here $\{p_t\}_{t=1}^{\infty}$ is a geometrically decreasing sequence, and therefore asset prices will converge much more quickly to their steady state values. It is easy to calculate that margins are still countercyclical:

$$m_t = 1 - \exp\left(-(1-\gamma)^{t-1}\right).$$

5 Policy measures

5.1 Margin requirements

It is sometimes argued that imposing margin requirements on asset-based borrowing would throw "sand in the wheels" in particular in the run-up of a bubble. Those policy measures can be analysed in the context of this model. A natural generalization of lemma 2.2 with margin requirements such that only a fraction m of this asset can be pledged is given by the following lemma. (the margin is therefore $1-m$)

Lemma 5.1 (Lending, no short-sales with margin requirements $1-m$). *There exist two cutoff expectations for future prices p_t and p_t'' with agents i satisfying $\mathbb{E}^i p_1 \leq (1+R')p_t$ invest in the storage technology ("pessimists"), agents i satisfying $\mathbb{E}^i p_1 \in [(1+R')p_t, (1+R')p_t'']$ lending to optimists ("moderates"), and agents i satisfying $\mathbb{E}^i p_1 \geq (1+R')p_t''$ investing in the bubble asset ("optimists"). These cutoff prices satisfy:*

$$\frac{1-F(p_t'')}{p_t} = \frac{S}{w_t} \tag{8a}$$

$$\frac{F(p_t)-F(mp_t)}{mp_t} + \int_{p_t''}^{mp_t} \frac{f(x)}{x} dx = \frac{S}{w_t} \tag{8b}$$

Proof. As with lending and no short-sales, there are two conditions reflecting the scarcity of resources to transfer and the scarcity of collateral.

- The market clearing for value of bubbles is now familiar and is unchanged:

$$\int_{p_t''}^{\frac{w_t}{S}} w_t f(x)dx = p_t S.$$

- The collateral constraint reflects the scarcity of asset collateral. Similarly as previously, only S bubbles can be pledged as collateral for loans. However, note that the most optimist lenders can no longer take too risky positions. That is, they must take at least $w_t/(mp_t)$ bubbles as collateral for their loans, while they would like to make more risky bets. Hence:

$$\int_{p_t''}^{mp_t} \underbrace{\frac{w_t}{x}}_{\text{market \# bubbles.}} f(x)dx + \int_{mp_t}^{p_t} \underbrace{\frac{w_t}{mp_t}}_{\text{regulatory \# bubbles.}} f(x)dx = \underbrace{S}_{\text{Total \# of bubbles}}.$$

□

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, I calculate that:

$$p_t(m) = \frac{1}{1+\mu(m)} \frac{w_t}{S} \qquad p_t''(m) = \frac{\mu(m)}{1+\mu(m)} \frac{w_t}{S}$$

with

$$\mu(m) = \frac{1}{m} e^{-\frac{1-m}{m}}.$$

Proof. See Appendix 2.1. $\square$

As can be seen on Figure 7, imposing margins is effective in reducing asset prices and slowing the buildup of a bubble. Interestingly, this intuition can however be reversed when short-sales are allowed. Because imposing margins prevent short-sellers from taking risks to sell the asset short, prices can be higher in equilibrium with the imposition of a margin requirement. I show below that it is the case for flat priors.

Lemma 5.2 (Lending, short-sales with margin requirements $1-m$)**.** *With both lending and short-sales, pessimists sell-short, negative moderates lend cash to optimists, positive moderates lend the asset to pessimists, and optimists buy the real asset. The cutoffs are such that:*

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \tag{9a}$$

$$\int_0^{p_t''} f(x)dx = \frac{1}{p_t}\int_{p_t/m}^{p_t'} (x-p_t)f(x)dx + \frac{1-m}{m}\left(\frac{F\left(\frac{p_t}{m}\right)-F(p_t)}{p_t}\right) \tag{9b}$$

$$\frac{F(p_t)-F(mp_t)}{mp_t} + \int_{p_t''}^{mp_t} \frac{f(x)}{x}dx = \frac{S}{w_t} \tag{9c}$$

Proof. The same logic as in the previous proof applies for the market clearing for resources and the equation reflecting market clearing for collateral. Note however that short-selling is now also more difficult because optimists lending the security (or alternatively buying replicas) now cannot take as much risk, and they must always keep a minimum guarantee of $p_t/m - p_t$ as cash collateral. Therefore:

$$\int_0^{p_t''} f(x)dx = \frac{1}{p_t}\int_{p_t/m}^{p_t'} (x-p_t)f(x)dx + \frac{1}{p_t}\int_{p_t}^{p_t/m}\left(\frac{p_t}{m}-p_t\right)f(x)dx.$$

$\square$

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, I calculate that:

$$p_t(m) = \frac{1}{\sqrt{2}\left(\frac{\lambda(m)}{\sqrt{e}}+\sqrt{2}\right)}\frac{w_t}{S} \qquad p_t'(m) = \frac{1}{\sqrt{2}}\frac{\sqrt{2}\lambda(m)+\sqrt{e}}{\lambda(m)+\sqrt{2e}}\frac{w_t}{S} \qquad p_t''(m) = \frac{m\exp\left(\frac{1-m}{m}\right)}{e\sqrt{2}\left(\frac{\lambda(m)}{\sqrt{e}}+\sqrt{2}\right)}\frac{w_t}{S}$$

with

$$\lambda(m) = \sqrt{m\exp\left(\frac{1-m}{m}\right) - \frac{e}{2}\left(\frac{1-m}{m}\right)^2}.$$

5.2 Monetary Policy

The model allows to study the impact of monetary policy on asset prices. In the case of dynamic inefficiency, it provides a new mechanism by which monetary policy can boost consumption, business or residential investment. This mechanism has the flavor of a "credit channel" for the transmission of monetary policy ; and more precisely, following the distinction made in **?**, that of a bank lending channel. Instead of relying on information asymmetries to drive a wedge between external sources of finance and internal one, this explanation relies on differences of opinion. However here borrowers can be both optimists and pessimists.

Contrary to **?** among others, it therefore may matter whether the central bank operates through conventional or unconventional monetary policy (at least if it buys assets which can be shorted) :

- In the case of conventional monetary policy, the central banks sets the fraction of deposits held as reserves by banks, or equivalently targets their price through the Fed funds rate for example. This effectively raises or decreases the implicit cost of deposit insurance, which banks do not pay in its entirety if the central bank provides liquidity in crisis times. In the model, this would show up as an implicit subsidy (or tax, when interest rates are high) to short-term loans.

- Unconventional monetary policy is perhaps even easier to understand through the lens of this model. In the case of long term assets, the central bank then accepts to refinance distressed borrowers using impaired collateral as a guarantee, with lower haircuts than those required by the market (through repos for example). It can also buy commercial paper requiring a preferential interest rate to refinance troubled institutions. In the model, this is akin to the introduction to a large player (a mass in the $f(.)$ density function) relatively optimistic relative to the rest of the market.

Unconventional monetary policy therefore has arguably less ambiguous effects on asset prices, as it does not help short-sellers as much as conventional monetary policy does. Another take from this analogy is that even if most of the financial sector which creates inside money is no longer regulated by the federal reserve, monetary policy can still be effective in crisis times to supply the market with optimistic beliefs.[21]

5.3 Currency crises

The model also helps speak to a central bank's optimal response when it wants to defend a peg against the attack of speculator. The model shows the existence of two opposite forces. On the one hand, raising interest rates increases the opportunity cost of not holding the currency (so even more that of shorting it) ; on the other, raising interest rates also make it more costly for optimists to leverage and bid up the targeted currency. Whether one force or the other is stronger again depends on the initial distribution of beliefs; I think my model is the first to generate this kind of predictions.

In the context of the Eurozone crisis, the model helps speak to the kind of interventions central banks may want to promote. In particular, there is little auguing that small purchases of peripheral countries' bonds helped alleviate short-term financing concerns for small countries. Further investigation into this issue through the lens of this model is an interesting topic for future research.

[21] Note that in the model, liquidity and solvency concerns are to a large extent concomittant. Defaults mount (wiping out capital set aside for regulatory purposes) as the same time as lending volumes fall.

6 Conclusion

This article has developed a Geanakoplos (1997) model of asset-based borrowing, allowing for short-sales as in Simsek (2012). Removing uncertainty and letting agents disagree about certain means rather than about probability of default states allows to keep a convex set of beliefs, and therefore to study the dynamics of these beliefs in a tractable way.

The model helps understand how leverage leads to a progressive build up in optimism, along with a decrease in margins required by lenders. Margins are also countercyclical when asset prices decrease following a relaxation of short-sales (margins then shoot up). Under some parameters value, the volume of lending is shown to shrink instantaneously; the volume of collateral used is discontinuous ; the model is therefore able to generate a phenomenon resembling the "run on repos" of 2007 documented in particular by Gorton and Metrick (2012). Moreover, following a relaxation of short-sales constraints, prices first decrease discontinuously but then continue to decrease as time goes on. Once again, to the best of my knowledge, this model is the first to be able to generate such protracted slumps.

In the model, agents are certain about the future, but they cannot be all right since they have different beliefs. An advantage of that is that I am not restricting the set of contracts exogenously, as in Geanakoplos (2003) and Simsek (2012), who study simple debt contracts (or simple short-sales contracts, in Simsek (2012)). Simple debt and short-sales contracts are the only available contracts, because there is no uncertainty. Another interpretation is that all agents are subject to a stochastic noise around this distribution, but that there is no point in contracting upon them since they are all correlated. Debt contracts may be optimal because agents disagree on means of distributions, not on their distribution functions. It would however be interesting to extend this analysis to the case where agents also agree to disagree on these probability changes.

In the multiple equilibria interpretation, where agents disagree on asset prices because they do not know how agents will coordinate next period, the adaptive learning studied here provides an equilibrium selection device for the location of rational bubbles. Rational bubbles therefore have the tendency to appear on assets which are easier to leverage against, and harder to sell-short; which may explain the prominent role played by real estate in financial crises, outlined in Reinhart and Rogoff (2008).

Finally, default is generally avoided in this model because prices increase when leverage is possible but short-sales are not. However, following a relaxation of short-sales for example, default spike despite complete markets: those are only a consequence of the fact that lenders were too optimistic about asset prices and did not ask for enough collateral. Interestingly, defaults mount as prices progressively decrease after the initial sharp fall in asset prices.

A recurring question is whether public authorities should try and manage those run ups in asset prices. Whether overoptimism about asset prices is good or bad amounts to determining of the economy is in a dynamically inefficient state or not. When it is not, public authorities should try and manage optimism but there is a question about whether public authorities have some superior ability than the market to do that. To the extent that such paternalism is desirable, Section 5 has shown that margin requirements were not necessarily an effective tool when short-sales are also possible. In contrast, if the economy is dynamically inefficient, then the formation of rational bubbles is indeed optimal and public authorities should not regulate the amount of leverage in the financial system. However, if one thinks that rational bubbles are very prone to reversals, there can still be a role for public policy. As is well known at least since Samuelson (1958) and Diamond (1965), public debt is also in the toolkit of public authorities, but these rational bubbles can be subject to the same kind of crises of confidence. The present analysis suggests that allowing more leverage on those assets could increase their appeal as a vehicle for transferring resources (for example by reducing capital requirements for holding public debt). Restricting short-sales on those instruments could have the same effect. I leave these very important questions for future research.

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1 More general short-sales possibilities

I have considered in the main text a case where lenders of securities do worry that they might not be repaid by short-sellers, but where short-sellers do not need to worry about the quality of their lenders, since lending operates through real assets. (so the lender can only lend w_t/p_t. However, given the specifics of the crisis in $2008-2009$, it can prove interesting to study what happens in two other configurations: one is the other extreme where short-sellers assess the quality of their counterparty (a trait of the 2008 financial crisis, according to Lewis (2010) in particular), and one is a middle range between the two previous extremes where both parties are equally worried about the other being able to honor his contract.

1.1 Endogenous short-sales constraints

"Lender's" creditworthiness

Lemma 1.1. *There exist two cutoff expectations for future prices p_t and p_t' with agents i satisfying $\mathbb{E}^i p_1 \leq (1+R')p_t'$ shorting the asset ("pessimists"), agents i satisfying $\mathbb{E}^i p_1 \in [(1+R')p_t', (1+R')p_t]$ being long the asset through replicas - equivalently, lending the asset to pessimists - ("moderates") , and agents i satisfying $\mathbb{E}^i p_1 \geq (1+R')p_t$ investing in the real asset ("optimists"). These cutoff prices satisfy:*

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \tag{10a}$$

$$\int_{p_t}^{p_t'} f(x)dx = \frac{1}{p_t}\int_0^{p_t}(p_t-x)f(x)dx. \tag{10b}$$

Proof. Once again, market clearing for bubbles says that optimists' endowments are invested in the bubble asset:

$$\int_{p_t'}^{\frac{w_t}{S}} w_t f(x)dx = p_t S.$$

The second equation changes however, because now short-sellers are worried about the quality of their counterparty, so that a short-seller with expectations x asks $p_t - x$ as collateral per unit of asset held. However, it is assumed here that lenders of security ask for the total value of the short-sale to stay with the borrower, so that borrowers can only borrow w_t/p_t with their endowments.

$$\int_0^{p_t} \underbrace{\frac{w_t}{p_t}}_{\#\text{ each holds}} (p_t - x)f(x)dx = w_t\int_{p_t}^{p_t'} f(x)dx.$$

$\square$

In that case, the number of synthetic contracts sold is:

$$S' = \frac{w_t}{p_t}F(p_t).$$

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that:

$$p_t = \frac{2}{5}\frac{w_t}{S} > \frac{1}{\sqrt{2}\left(1+\sqrt{2}\right)}\frac{w_t}{S} \qquad p'_t = \frac{3}{5}\frac{w_t}{S} > \frac{1}{\sqrt{2}}\frac{w_t}{S} \qquad S' = \frac{5}{2}S.$$

Symmetric case

Lemma 1.2. *There exist two cutoff expectations for future prices p_t and p'_t with agents i satisfying $\mathbb{E}^i p_1 \leq (1+R')p'_t$ shorting the asset ("pessimists"), agents i satisfying $\mathbb{E}^i p_1 \in [(1+R')p'_t, (1+R')p_t]$ being long the asset through replicas - equivalently, lending the asset to pessimists - ("moderates") , and agents i satisfying $\mathbb{E}^i p_1 \geq (1+R')p_t$ investing in the real asset ("optimists"). These cutoff prices satisfy:*

$$\frac{1-F(p'_t)}{p_t} = \frac{S}{w_t} \tag{11a}$$

$$p_t = \frac{1}{F(p'_t)}\int_0^{p'_t} xf(x)dx \tag{11b}$$

Proof. Once again, market clearing yields:

$$\int_{p'_t}^{\frac{w_t}{S}} w_t f(x)dx = p_t S.$$

Equation (11b) requires a bit more work. Denoting by S' the total number of contracts bought/sold by both parties, I can write two equations corresponding to the two polar cases outlined before, except that S' was $w_t/p_t(F(p'_t) - F(p_t))$ in the case where lenders were assessing the quality of their counterparty, and $w_t/p_t F(p_t)$ in the opposite case. In this intermediate case, S' appears in two equations :

$$\int_{p_t}^{p'_t} \frac{S'}{F(p'_t)-F(p_t)}(x-p_t)f(x)dx = \int_0^{p_t} w_t f(x)dx$$
$$\int_0^{p_t} \frac{S'}{F(p_t)}(p_t-x)f(x)dx = \int_{p_t}^{p'_t} w_t f(x)dx.$$

□

Dividing one equation by the other and simplifying yields:

$$\int_{p_t}^{p'_t}(x-p_t)f(x)dx = \int_0^{p_t}(p_t-x)f(x)dx$$
$$\Leftrightarrow \quad p_t = \frac{1}{F(p'_t)}\int_0^{p'_t} xf(x)dx$$

One can use one equation or the other to calculate the number of short contracts:

$$S' = \frac{F(p_t)}{\frac{1}{F(p'_t)-F(p_t)}\left[\int_{p_t}^{p'_t} xf(x)dx\right] - p_t} = \frac{F(p'_t)-F(p_t)}{p_t - \frac{1}{F(p_t)}\left[\int_0^{p_t} xf(x)dx\right]}$$

In particular, for the case of flat priors, that is, $f(x) = S/w_t$, one gets that (after simple but somewhat tedious computations for S') :

$$\frac{1}{\sqrt{2}\,(1+\sqrt{2})}\frac{w_t}{S} < p_t = \frac{1}{3}\frac{w_t}{S} < p_t = \frac{2}{5}\frac{w_t}{S}$$
$$p'_t = \frac{2}{3}\frac{w_t}{S} \quad \Rightarrow \quad S' = 2S.$$

1.2 Exogenous short-sales constraints

Once again, I have assumed in the main text that lenders needed to have full ownership of the asset to lend it away. I investigate here alternative arrangements for short-selling. First, I look what happens when collateral must stay with the lender (that is, lenders are fully backed). I then study the intermediate case where both lenders and borrowers ask to the other party the minimum collateral consistent with their belief.

"Lender's" creditworthiness

In the case where collateral must stay with the lender, equilibrium is given by:

$$\frac{1-F(p'_t)}{p_t} = \frac{S}{w_t} \tag{12a}$$
$$\frac{F(p'''_t)}{p_t} = \frac{S'}{w_t} \tag{12b}$$
$$\int_{p_t}^{p'_t} f(x)dx = \frac{1}{p_t}\int_0^{p'''_t}(p_t - x)f(x)dx. \tag{12c}$$

Note that equation (12b) replaces equation (5b) in the main text, where S' was given by $w_t(F(p'_t) - F(p_t))/p_t$. Together with equation (5a), it gives equation (5b).

With flat priors, that is, $f(x) = S/w_t$, one calculates that:

$$p'''_t = \frac{S'}{S}p_t \quad \text{from equation (12b)}, \quad p_t + p'_t = \frac{w_t}{S} \quad \text{from equation (12a)}$$
$$\int_{p_t}^{p'_t} f(x)dx = \frac{1}{p_t}\int_0^{p'''_t}(p_t - x)f(x)dx \quad \Rightarrow \quad p_t(p'_t - p_t) = \frac{p_t^2}{2} - \frac{(p_t - p'''_t)^2}{2}$$
$$\Rightarrow \quad p_t\left(\frac{w_t}{S} - 2p_t\right) = \frac{1}{2}\left[\frac{2S'}{S}p_t^2 - \left(\frac{S'}{S}\right)^2 p_t^2\right]$$
$$\Rightarrow \quad p_t = \frac{1}{2 + \frac{S'}{S} - \frac{S'^2}{2S^2}}\frac{w_t}{S} \quad \Rightarrow \quad p'''_t = \frac{\frac{S'}{S}}{2 + \frac{S'}{S} - \frac{S'^2}{2S^2}}\frac{w_t}{S} \quad \Rightarrow \quad p'_t = \frac{1 + \frac{S'}{S} - \frac{S'^2}{2S^2}}{2 + \frac{S'}{S} - \frac{S'^2}{2S^2}}\frac{w_t}{S}.$$

Symmetric case

In the symmetric case where buyers and sellers of synthetic securities are equally worried about the creditworthiness of their counterparties, the cutoffs are given by the following set of equations:

$$\int_{p_t}^{p'_t} \frac{S'}{F(p'_t) - F(p_t)}(x - p_t)f(x)dx = \int_0^{p'''_t} w_t f(x)dx$$
$$\int_0^{p'''_t} \frac{S'}{F(p'''_t)}(p_t - x)f(x)dx = \int_{p_t}^{p'_t} w_t f(x)dx$$
$$\frac{1 - F(p'_t)}{p_t} = \frac{S}{w_t}.$$

In the case with flat priors, the three equations rewrite simply:

$$\int_{p_t}^{p'_t} \frac{S'}{F(p'_t) - F(p_t)}(x - p_t)f(x)dx = \int_0^{p'''_t} w_t f(x)dx \quad \Rightarrow \quad S'(p'_t - p_t) = Sp'''_t$$
$$\int_0^{p'''_t} \frac{S'}{F(p'''_t)}(p_t - x)f(x)dx = \int_{p_t}^{p'_t} w_t f(x)dx \quad \Rightarrow \quad S'\left[p_t - \frac{p'''_t}{2}\right] = S(p'_t - p_t)$$
$$\frac{1 - F(p'_t)}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad p_t + p'_t = \frac{w_t}{S}.$$

Solving this linear system of 3 equations and 3 unknowns yields:

$$p'_t = \left(\frac{1}{2} + \frac{S^2}{S'^2} + \frac{S}{S'}\right)p'''_t \quad \text{and} \quad p_t = \left(\frac{1}{2} + \frac{S^2}{S'^2}\right)p'''_t.$$

Hence:

$$p'''_t = \frac{1}{1 + 2\frac{2S^2}{S'^2} + \frac{S}{S'}} \quad \Rightarrow \quad p_t = \frac{\frac{1}{2} + \frac{S^2}{S'^2}}{1 + 2\frac{2S^2}{S'^2} + \frac{S}{S'}} \quad \Rightarrow \quad p'_t = \frac{\frac{1}{2} + \frac{S^2}{S'^2} + \frac{S}{S'}}{1 + 2\frac{2S^2}{S'^2} + \frac{S}{S'}}.$$

2 Omitted proofs

2.1 Static prices - priors calculations

I will provide here some sketch of calculations for the closed-form flat priors solutions.

No lending, no short-sales

The case with no lending, no short sales is immediate from equation (1):

$$\frac{1 - F(p_t)}{p_t} = \frac{S}{w_t} \quad \Leftrightarrow \quad 1 - \frac{S}{w_t}p_t = \frac{S}{w_t}p_t \quad \Leftrightarrow \quad p_t = \frac{w_t}{2S}.$$

Lending, no short-sales

The case with lending and no short-sales is also a straightforward system with two equations and two unknowns. In particular, from (2b) one gets:

$$\int_{p_t''}^{p_t} \frac{f(x)}{x} dx = \frac{S}{w_t} \quad \Leftrightarrow \quad p_t = ep_t''.$$

Plugging in (2a) gives :

$$\frac{1 - F(p_t'')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad 1 - \frac{S}{w_t}\frac{p_t}{e} = \frac{S}{w_t}p_t \quad \Rightarrow \quad p_t = \frac{e}{e+1}\frac{w_t}{S} \quad \Rightarrow \quad p_t'' = \frac{1}{e+1}\frac{w_t}{S}.$$

No lending, short-sales

With no lending and short-sales there are again two unknowns and two equations. Using first in equation (3b):

$$\int_0^{p_t} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'} (x - p_t)f(x)dx \quad \Rightarrow \quad p_t = \frac{1}{p_t}\frac{(p_t' - p_t)^2}{2} \quad \Rightarrow \quad p_t' = (\sqrt{2} + 1)p_t.$$

The last implication follows from the fact that $p_t' > p_t$ because the marginal buyers of replicas have a lower valuation than the marginal buyers of real assets. Then, replacing in equation (3a):

$$\frac{1 - F(p_t')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad 1 - \frac{S}{w_t}p_t' = \frac{S}{w_t}p_t \quad \Rightarrow \quad p_t = \frac{1}{\sqrt{2}\left(1 + \sqrt{2}\right)}\frac{w_t}{S} \quad \Rightarrow \quad p_t' = \frac{1}{\sqrt{2}}\frac{w_t}{S}.$$

To calculate the number of synthetic contracts, use:

$$S' = \frac{w_t}{p_t}(F(p_t') - F(p_t)) \quad \Rightarrow \quad S' = \sqrt{2}(1 + \sqrt{2})\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}(1 + \sqrt{2})}\right] \quad \Rightarrow \quad S' = \sqrt{2}S.$$

Lending, short-sales

With lending and short-sales, there are three unknowns to solve for. Equation (4c) gives:

$$\int_{p_t''}^{p_t} \frac{f(x)}{x} dx = \frac{S}{w_t} \quad \Rightarrow \quad p_t = ep_t''.$$

Equation (4b) gives:

$$\int_0^{p_t''} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'} (x - p_t)f(x)dx \quad \Rightarrow \quad p_t'' = \frac{1}{p_t}\frac{(p_t' - p_t)^2}{2} \quad \Rightarrow \quad p_t' = \frac{\sqrt{2} + \sqrt{e}}{\sqrt{e}}p_t$$

Equation (4a) gives:

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad p_t' + p_t = \frac{w_t}{S} \quad \Rightarrow \quad p_t = \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{e}} + \sqrt{2}\right)} \frac{w_t}{S} \quad \Rightarrow \quad p_t' = \frac{1}{\sqrt{2}} \frac{\sqrt{2}+\sqrt{e}}{1+\sqrt{2e}} \frac{w_t}{S}$$
$$\Rightarrow \quad p_t'' = \frac{1}{e\sqrt{2}\left(\frac{1}{\sqrt{e}} + \sqrt{2}\right)} \frac{w_t}{S}$$

Calculating the number of synthetic contracts again comes from:

$$S' = \frac{w_t}{p_t}\left[F(p_t') - F(p_t)\right] \quad \Rightarrow \quad S' = \sqrt{2}\left(\frac{1}{\sqrt{e}} + \sqrt{2}\right)\left[\frac{1}{\sqrt{2}} \frac{\sqrt{2}+\sqrt{e}}{1+\sqrt{2e}} - \frac{\sqrt{e}}{\sqrt{2}(1+\sqrt{2e})}\right] \quad \Rightarrow \quad S' = S.$$

No lending, moderate short-sales

Let's now turn to the case of moderate short-sales. With moderate short-sales (of maximum size S') and no lending, equation (5b) gives:

$$\frac{1-F(p_t)}{p_t} = \frac{S+S'}{w_t} \quad \Rightarrow \quad p_t = \frac{S}{2S+S'} \frac{w_t}{S}.$$

Equation (5a) yields:

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad 1 - \frac{S}{w_t} p_t' = \frac{S}{w_t} p_t \quad \Rightarrow \quad p_t' = \frac{S+S'}{2S+S'} \frac{w_t}{S}.$$

Finally, the threshold for short-sales is given by equation (5c):

$$\int_0^{p_t'''} f(x)dx = \frac{1}{p_t} \int_{p_t}^{p_t'} (x-p_t) f(x) dx \quad \Rightarrow \quad p_t''' = \frac{1}{p_t} \frac{(p_t'-p_t)^2}{2} \quad \Rightarrow \quad p_t''' = \frac{S'^2}{2S(2S+S')} \frac{w_t}{S}.$$

Lending, moderate short-sales

Finally, with moderate short-sales (of size S') and lending, equation (6d) gives:

$$\int_{p_t''}^{p_t} \frac{f(x)}{x} dx = \frac{S}{w_t} \quad \Rightarrow \quad \ln\left(\frac{p_t}{p_t''}\right) = 1 \quad \Rightarrow \quad p_t = e p_t''.$$

From equation (6b):

$$\frac{1-F(p_t'')}{p_t} = \frac{S+S'}{w_t} \quad \Rightarrow \quad p_t = \frac{e}{1+\frac{S+S'}{S}e} \frac{w_t}{S} \quad \Rightarrow \quad p_t'' = \frac{1}{1+\frac{S+S'}{S}e} \frac{w_t}{S}$$

Equation (6a) then enables to calculate p_t':

$$\frac{1-F(p_t')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad p_t' = \frac{1+\frac{S'}{S}e}{1+\frac{S+S'}{S}e} \frac{w_t}{S}.$$

Finally, equation (6c) gives p_t''':

$$\int_0^{p_t'''} f(x)dx = \frac{1}{p_t}\int_{p_t}^{p_t'}(x-p_t)f(x)dx \quad \Rightarrow \quad p_t''' = \frac{1}{p_t}\frac{(p_t'-p_t)^2}{2} \quad \Rightarrow \quad p_t''' = \frac{1}{2e}\frac{\left(1+\frac{S'-S}{S}e\right)^2}{1+\frac{S+S'}{S}e}\frac{w_t}{S}.$$

2.2 Policy - flat priors calculations

Margin requirements without short-sales

If a borrower can only borrow up to a fraction m of the asset, then given m there is a system of two equations with two unknowns.[22] In particular, from (8b) one gets:

$$\frac{F(p_t)-F(mp_t)}{mp_t} + \int_{p_t''}^{mp_t}\frac{f(x)}{x}dx = \frac{S}{w_t} \quad \Leftrightarrow \quad \frac{p_t - mp_t}{mp_t} + ln\left(\frac{mp_t}{p_t''}\right) = 1$$
$$\Leftrightarrow \quad p_t'' = \frac{m}{e}\exp\left(\frac{1-m}{m}\right)p_t.$$

Plugging in (8a) gives :

$$\frac{1-F(p_t'')}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad 1 - \frac{S}{w_t}\frac{m}{e}\exp\left(\frac{1-m}{m}\right)p_t = \frac{S}{w_t}p_t \quad \Rightarrow \quad p_t = \frac{e}{e+m\exp\left(\frac{1-m}{m}\right)}\frac{w_t}{S}$$
$$\Rightarrow \quad p_t'' = \frac{m\exp\left(\frac{1-m}{m}\right)}{e+m\exp\left(\frac{1-m}{m}\right)}\frac{w_t}{S}.$$

Margin requirements with short-sales

Given margin requirements $1-m$, there are three unknowns to solve for. Equation (9c) gives:

$$\frac{F(p_t)-F(mp_t)}{mp_t} + \int_{p_t''}^{mp_t}\frac{f(x)}{x}dx = \frac{S}{w_t} \quad \Rightarrow \quad \frac{p_t - mp_t}{mp_t} + \ln\left(\frac{mp_t}{p_t''}\right) = 1$$
$$\Rightarrow \quad p_t'' = \frac{m}{e}\exp\left(\frac{1-m}{m}\right)p_t.$$

Equation (9b) gives:

$$\int_0^{p_t''} f(x)dx = \frac{1}{p_t}\int_{p_t/m}^{p_t'}(x-p_t)f(x)dx + \frac{1-m}{m}\left(\frac{F\left(\frac{p_t}{m}\right)-F(p_t)}{p_t}\right)$$
$$\Rightarrow \quad p_t'' = \frac{1}{p_t}\frac{(p_t'-p_t)^2}{2} + \frac{1}{p_t}\frac{\left(\frac{p_t}{m}-p_t\right)^2}{2}$$
$$\Rightarrow \quad \frac{2m}{e}\exp\left(\frac{1-m}{m}\right)p_t^2 = (p_t'-p_t)^2 + \left(\frac{p_t}{m}-p_t\right)^2.$$

Equation (9a) gives:

[22] I drop dependence on m for lighter notation.

$$\frac{1-F(p'_t)}{p_t} = \frac{S}{w_t} \quad \Rightarrow \quad p'_t + p_t = \frac{w_t}{S} \quad \Rightarrow \quad \left[\frac{2m}{e}\exp\left(\frac{1-m}{m}\right) - \left(\frac{1-m}{m}\right)^2\right] p_t^2 = \left(\frac{w_t}{S} - 2p_t\right)^2$$

For sufficiently low margin requirements $1-m > 1/2$ which implies that the quantity in the square brakets is positive,

$$\left(\sqrt{\frac{2m}{e}\exp\left(\frac{1-m}{m}\right) - \left(\frac{1-m}{m}\right)^2} + 2\right) p_t = \frac{w_t}{S} \quad \Rightarrow \quad p_t = \frac{1}{\sqrt{2}\left(\frac{\lambda(m)}{\sqrt{e}} + \sqrt{2}\right)} \frac{w_t}{S}$$
$$\Rightarrow \quad p'_t = \frac{1}{\sqrt{2}} \frac{\sqrt{2}\lambda(m) + \sqrt{e}}{\lambda(m) + \sqrt{2e}} \frac{w_t}{S} \quad \Rightarrow \quad p''_t = \frac{m\exp\left(\frac{1-m}{m}\right)}{e\sqrt{2}\left(\frac{\lambda(m)}{\sqrt{e}} + \sqrt{2}\right)} \frac{w_t}{S}.$$

With $\lambda(.)$ defined over $[0, 1/2]$ as:

$$\lambda(m) = \sqrt{m\exp\left(\frac{1-m}{m}\right) - \frac{e}{2}\left(\frac{1-m}{m}\right)^2}.$$

2.3 Other proofs

No short-sales, lending correspondence

As shown in the main text, reasoning in terms of margins yields:

$$\begin{aligned}
\frac{1}{F(p_t) - F(p''_t)} \int_{p''_t}^{p_t} w_t \frac{p_t - x}{x} f(x)dx &= w_t \frac{1 - F(p_t)}{F(p_t) - F(p''_t)} \\
\Leftrightarrow \quad \int_{p''_t}^{p_t} \frac{p_t - x}{x} f(x)dx &= 1 - F(p_t) \\
\Leftrightarrow \quad \int_{p''_t}^{p_t} \frac{f(x)}{x} dx &= \frac{1 - F(p''_t)}{p_t} \\
\Leftrightarrow \quad \int_{p''_t}^{p_t} \frac{f(x)}{x} dx &= \frac{S}{w_t} \quad \text{using equation (2a).}
\end{aligned}$$

Reasoning in terms of margins or collateral therefore yields the same results.

Proof of Lemma 3.1

The correspondence is immediate from:

$$F_t(X) = \mathbb{P}(p^e_{t+1} \leq X) = \mathbb{P}((1-\gamma_t)p^e_t + \gamma_t p_{t-1} \leq X) = F_{t-1}\left(\frac{X - \gamma_t p_{t-1}}{1 - \gamma_t}\right).$$

No lending, short-sales with econometric learning

As is shown in the main text, equilibrium conditions give a quadratic expression for prices p_t:

$$\left[(a_t+1)^2-2\right]p_t^2-2a_t(a_t+b_t+1)p_t+(a_t+b_t)^2=0.$$

For this equation, the discriminant is:

$$\Delta=4a_t^2(a_t+b_t+1)^2+4(a_t+b_t)^2[2-(a_t+1)^2].$$

$2-(a_t+1)^2\geq 0$ is a sufficient condition for $\Delta\geq 0$. This condition is equivalent to $a_t\leq\sqrt{2}-1$ which is true whenever $t\geq 4$ because $a_t=\frac{1}{t-1}$.

Lending, short-sales

$$\begin{aligned}\Delta\leq 0\quad&\Leftrightarrow\quad a_t(a_t+b_t+1)\leq(a_t+b_t)\sqrt{(a_t+1)^2-2e^{-a_t}}\\ \Leftrightarrow\quad&(a_t+1-\sqrt{(a_t+1)^2-2e^{-a_t}})a_t\leq b_t\left(\sqrt{(a_t+1)^2-2e^{-a_t}}-a_t\right)\\ \Leftrightarrow\quad&b_t\geq\frac{a_t+1+\sqrt{(a_t+1)^2-2e^{-a_t}}}{\sqrt{(a_t+1)^2-2e^{-a_t}}-a_t}\quad\text{and}\quad(a_t+1)^2\leq a_t^2+2e^{-a_t}.\end{aligned}$$

The last equivalence follows from the fact that $a_t+1-\sqrt{(a_t+1)^2-2e^{-a_t}}\geq 0$.

Symmetric short-sales, econometric learning

From Appendix 1, with symmetric short-sales constraints (that is, equally disciplined by both short-sellers and owners of the synthetic asset):

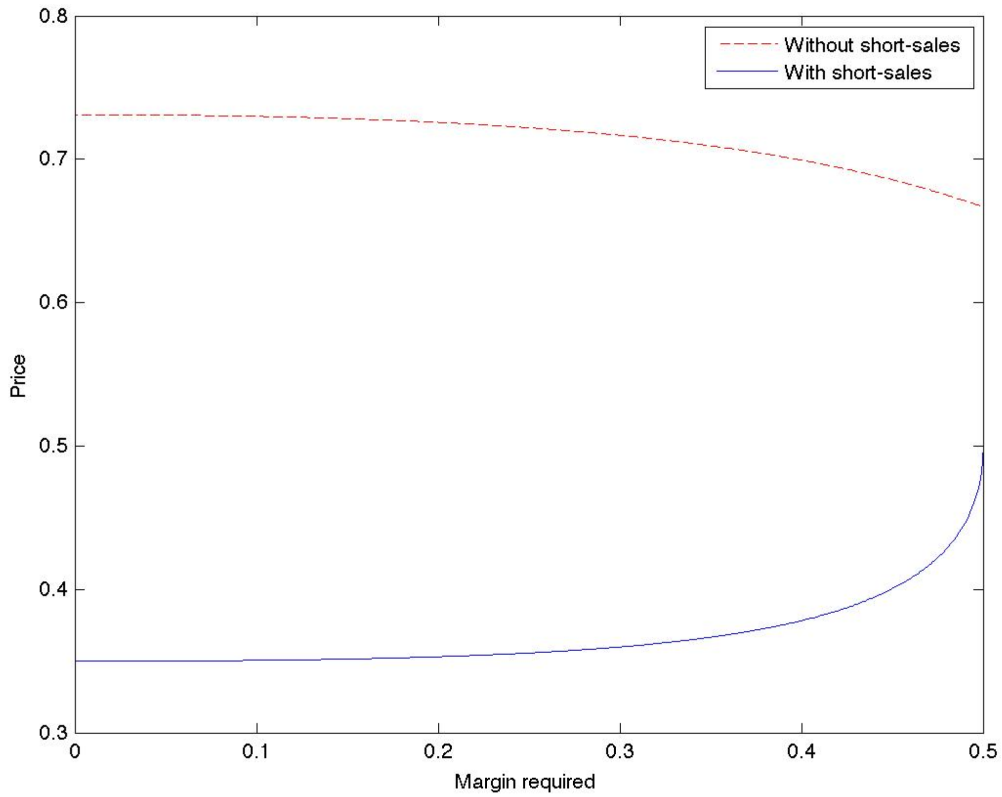
$$\begin{aligned}p_t=\frac{1}{F(p_t')}\int_0^{p_t'}xf(x)dx\quad&\Leftarrow\quad p_t=\frac{a_t}{p_t'-b_t}\int_{b_t}^{p_t'}\frac{1}{a_t}xdx\\ &\Leftarrow\quad p_t=\frac{p_t'^2-b_t^2}{2(p_t'-b_t)}=p_t'+b_t.\end{aligned}$$

Together with the usual equation $p_t'=b_t+a_t(1-p_t)$ coming from market clearing for real assets, one has a system of two linear equations to solve:

$$p_t=1-\frac{2(1-b_t)}{2+a_t}\qquad p_t'=b_t+\frac{2a_t(1-b_t)}{2+a_t}$$

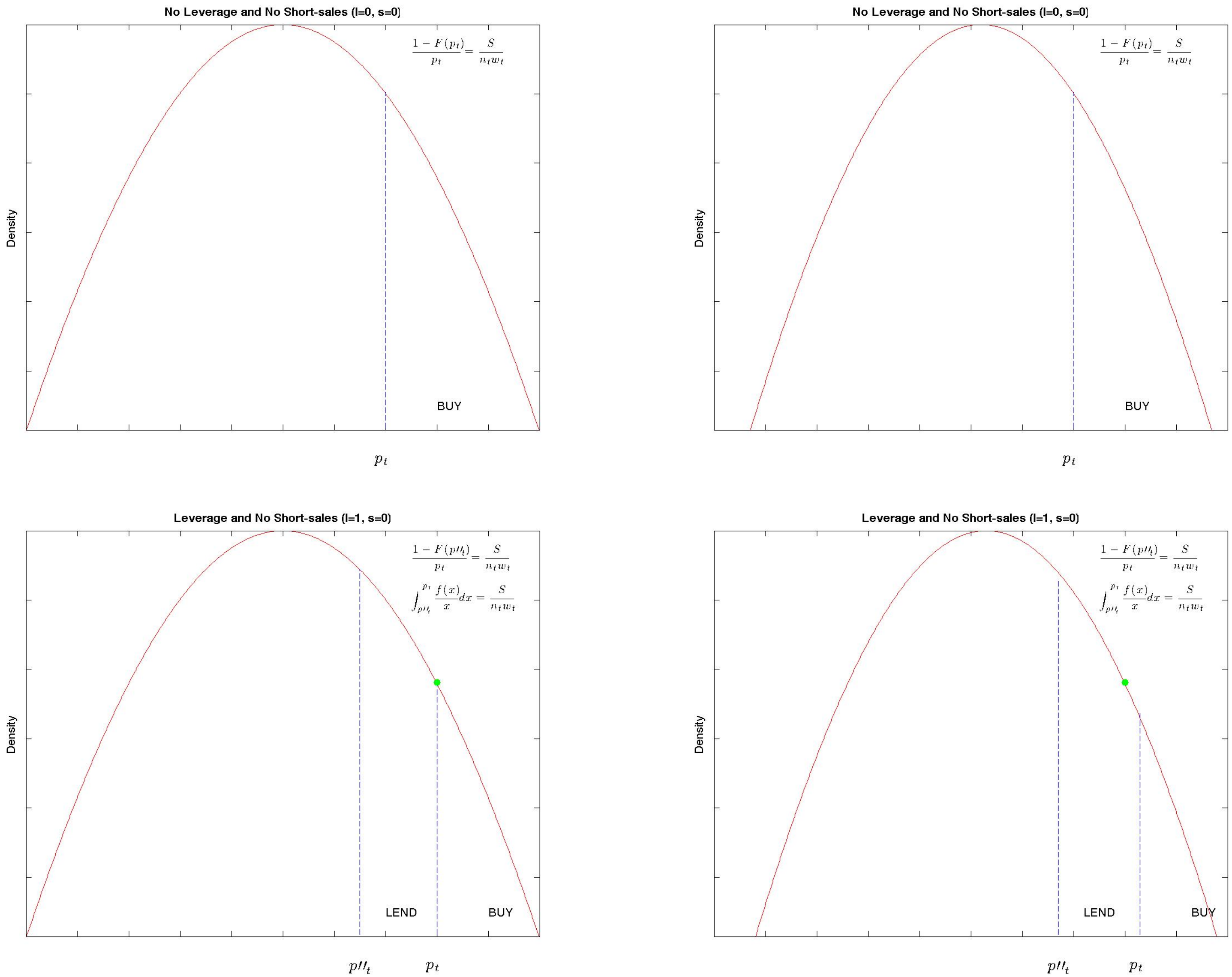
3 Figures

Figure 7: Effectiveness of margin requirements



Note: The x-axis represents the minimum required level of margins. The y-axis gives the resulting price in the case of flat priors, that is $\forall x \in [0, 1], f(x) = 1$. Without short-sales constraints, margins have the intuitive effect of lowering asset prices. However, with short-sales constraints, margins here contribute to overoptimism. Intuitively, this is because margin also throw "sands in the wheels" of short-sellers.

Figure 8: Intuition: leverage and no leverage



Chapter 3

Rational Bubbles Driven Business Cycles

Abstract

I augment the Diamond (1965) neoclassical growth model with a labor/leisure margin and rational bubbles shocks. I show that those shocks can drive business cycles fluctuations, leading to higher consumption, investment and hours. Elasticities can be calibrated to be compatible with balanced growth, and microeconomic estimates for labor supply, while generating large fluctuations in hours. The model features countercyclical labor wedges even though wages are set competitively, that come from the decrease of interest rates during bubbles' busts. It can generate "jobless recoveries": the stock market can rebound with initially decreasing levels of employment.

Keywords: Business cycles, bubbles, jobless recoveries.

JEL classification: E24, E32, E44

Acknowledgements: I am very grateful to Emmanuel Farhi, Philippe Martin and Jonathan Parker for very helpful conversations. I also thank John Campbell, Roger Guesnerie, Guy Laroque and seminar participants at Harvard University for useful comments and suggestions. All errors remain mine. I am extremely grateful to "Corps des Ponts" for funding this research and to Harvard University for their hospitality.

Introduction

The Real Business Cycle literature has grown spectacularly over the years and is now the standard tools in macroeconomics for the study of economic fluctuations. Yet there is increasing skepticism, especially after the $2007-2009$ recession, that technology shocks can always be the major source of business cycle fluctuations. The biggest puzzle is that apart from clearly identified supply shocks (like the 1973 and 1979 oil shocks), it is hard to see technological recessions in the data; so that productivity shocks are more convincing at explaining booms than their subsequent busts.[1]

[1] Note that there already was some skepticism when King and Rebelo (1999) wrote the handbook chapter on Real Business Cycles.

This paper argues that a new, additional source of business cycles fluctuations can be added to the real business cycle model, which similarly induces consumption, hours and investment to comove positively : rational bubbles shocks. The broad mechanism at play in the model is the following. Agents do not just work for contemporaneous consumption, but also for their retirement consumption and for leaving bequests to their children. In this environment, not only their current wages matter for labor supply but also the return they will get when they consume their wages or bequest them. When agents become able to coordinate on a rational bubble, which I call a "rational bubble shock" (for example, as shown in Chapter 2), adaptive learning can play such a role as a coordination device), interest rates start to rise and agents start working more because they suddenly can enjoy more consumption when retired[2] or leave more money to their children.[3] Because they work more during these rational bubbles driven booms, and that factors of production are somewhat complementary in the production function, these rational bubbles can also crowd investment in, in addition to crowding it out by competing with capital as a store of value for savings. If the former effect is greater, investment is crowded in on average. Thus, introducing a labor/margin helps get rid of an important reason why rational bubbles theory has been dismissed, namely that they crowd out productive investment, which arguably does not "sound right". Importantly, although investment goes up in these cycles, the capital per hour worked (a counterpart of the capital per capita in Diamond (1965), where labor is supplied inelastically) goes down, driving up interest rates and driving down wages, which are countercyclical in such cycles.

Although the claim that business cycles could be driven by expectations about future prices rather than fundamentals will not convince all, I think it is worthwhile to go with the advantages of such an addition. The first advantage is that it is much easier to argue for completely unexpected or uninsurable rational bubbles shocks, because with dynamic inefficiency, there is a multiplicity of Rational Expectations Equilibria.[4] The second is that rational bubbles shocks are endogenously persistent, as shown in Chapter 2, while the persistence of productivity shocks needs to be assumed. For example, a one-off episode of financial liberal-

[2]Strictly speaking, there are really to effects of rational bubbles in Diamond (1965) overlapping generations model. The first is that they crowd out inefficient investment, making the storage technology more effective overall, and therefore yielding higher consumption when both old and young. The second is that they allow a better allocation of consumption between young and old age, even keeping the "total size of the pie" constant. In the Samuelson (1958) model, only the latter effect is at play.

[3]The reader might here wonder whether the possibility of bequests does not bring us back to the infinite horizon model, as suggested by Barro (1974). I argue that this is not the case as long as the motive for bequests does not stem from caring about their children's utility combined with impatience. Agents could very well leave bequests because of a "warm glow" of giving bequests : note that in the case of dynamic inefficiency where interest rates are low $r < g$ Barro (1974)-type "altruistic" parents would paradoxically borrow from their children. In other words, the need for having a well-defined maximization problem for altruistic families forces us to assume a high degree of impatience from their part, in turn restricting their levels of generosity. Notwithstanding the fact that bequests would be predicted to be negative in the current environment, such borrowing of parents from their children is forbidden (at least in all countries I know of) in the world where it is not possible to bequest debts.

[4]The sunspot literature, initiated in particular by Azariadis (1981), on contrary postulates the existence of a coordination device (a sunspot) whose occurrence is measurable. Without having a theory of what these sunspots are, it is perhaps safer to argue that multiple equilibria cannot be ruled out in every period, and that insurance is therefore impossible. In Chapter 2, I investigate adaptive learning as a potentially powerful coordination device for the location of rational bubbles. I show that the precise effect of financial liberalization shocks for example, cannot be perfectly anticipated with adaptive learning, and that prices under such contigencies are non contractible.

ization (like an increase in authorized Loan-To-Value ratio for home-buyers or a decrease in capital requirements for banks) or a one-off decrease in the central bank's policy rate (providing cheap funding for providers of asset-backed lending) is shown to lead to persistent and gradual appreciation in asset values. The third is that negative rational bubbles shocks are a feature of the data, and that they can be caused by identified changes in institutional environments (like a change in short-sales constraints), while it is hard to find such negative counterpart for productivity shocks. The fourth is that those shocks induce a negative co-movement of real wages with employment and output, which is a puzzling feature of the last cycles in the United States. This happens even though hours of work are chosen voluntarily: the decrease in real wage has a negative effect on employment but this is offset by the higher expected returns brought about by the bubble: those higher expected returns both follow from the crowding out of capital (raising interest rates) and from the expected capital gains on the bubble. Therefore, the model rationalizes the countercyclical labor wedge that is seen in the data: lowered interest rates do act as an implicit tax on labor during recessions. Interestingly, the burst of rational bubbles does not have the same effects depending on who were holding them and whose expectations were brought down: a fall of a housing bubble has a much bigger effect than that of a stock market one, for instance. Fifth, it suggests that the Frisch elasticity of labor supply might be much lower and closer to microeconomic estimates than previously thought. Sixth, consumption comoves positively with asset prices and negatively with expected returns. However, the mechanism at play here is not that of insurance as in the Consumption Capital Asset Pricing Model (for example). The fact that consumption falls relatively little compared to the rise in expected returns is not a puzzle in such a framework, as the comovement is not related to risk aversion in any way ; consumption rises when asset prices rise and expected returns go down because old agents are richer and consume more before they die. Seventh, long run real interest rates fluctuate in the model as in the data, without assuming preference shocks (impatience shocks for example): they are low in recessions, and high in booms.

To be more concrete, two crucial modifications I make in the canonical Real Business Cycle model are essential for my results. First, I replace the consumption Euler equation of the representative agent by a differential consumption of young and old agents. It is therefore no longer a transversality condition at infinity that determines long run interest rates for the steady state. As is well known in Diamond (1965) can be disconnected from the rate of time preference, completely independent of impatience, unlike in the Fisher (1930) theory. One over many advantages of this approach is that it is not contradicted by Mankiw et al. (1985)'s evidence that Euler equations do not seem to be a feature of the data. Moreover, preference shocks in the form of shocks to the rate of impatience of consumers have often be used as a means to generate realistic business cycle experiments, or falls in the real rate of interest in New-Keynesian models, but such shocks are not needed in a Diamond (1965) where interest rates evolve independently of impatience. Second, the way steady-state interest rates are determined in this class of models opens the possibility for overaccumulation of capital, or dynamic inefficiency (as in Diamond (1965)) and multiple rational expectations equilibria for asset prices coming from the possibility of rational bubbles. This is the second ingredient I will add to the Real Business Cycle models: agents will be allowed to "overvalue" assets in the sense of the present value discounted value of their fundamentals, and rational bubbles will compete for savings with productive capital.

The results obtained under the theoretical framework developed in the paper are very consistent with the data carefully assembled about the business cycles. For example, it has long been recognized that labor wedge shocks, due for example to changes in the tax code, could similarly lead to positive comovements between hours, investment, consumption and output. However, as King and Rebelo (1999) rightly pointed out, this explanation did not seem to square well with the fact that those changes of tax codes are very rare and that, apart from few notable exceptions (the Reagan tax cuts for example), few boom episodes could be associated with such observable changes in labor taxes. In the model I present here, real interest rates and expected capital gains on rational bubbles really act as a labor wedge, which varies with rational bubbles shocks.

Finally, I then extend the Diamond (1965) RBC model constructed above in two interesting dimensions. The first extension allows for costly reproduction of rational bubbles; I show that rational bubbles come with distortions in the production sector, because they encourage the production of assets to which they are attached. A second extension allows for two sectors one of which provides services for the old. The purpose of this extension is to show the potentially detrimental consequences of the rise and fall of rational bubbles: they do not only solve the dynamic inefficiency problem, but also come about with real distortive effets on the allocation of production.

Literature. The Real Business Cycles literature was started by the seminal work of Kydland and Prescott (1982), who argued that the bulk of United States fluctuations could be explained by a neoclassical growth model with competitive markets augmented with technology shocks and a labor/leisure margin. It is impossible to make justice to the whole RBC research program in this review of the literature, so I will only name a few of those articles. Gali (1999) shows that the RBC models is hard to reconcile with the empirical moments conditional of different types of shocks. Basu et al. (2006) provide evidence that technology shocks can actually be contractionary. Beaudry and Portier (2006) show that changes in expectations can very well be linked to fundamentals when they anticipate future changes on productivity. Beaudry and Portier (2007) show that positive comovements between output, consumption and employment cannot be driven by news shocks to future productivities in one good model. The model presented here is different in two different aspects: rational bubbles have no fundamental element to them, and they therefore only represent good news about the efficiency of the savings technology, and I use a model with finite lives. Grandmont (1985) shows that business cycles can be generated in perfectly deterministic competitive OLG models. Finally, Farhi and Tirole (2011) show that the need for liquidity can create a demand for stores of value that rational bubbles fulfil. In contrast, I use a perfectly neoclassical setting and also get positive comovement with employment. The idea of incorporating a labor/leisure margin in OLG models to generate business cycles is hardly a new one. It was already present in Azariadis (1981), Grandmont (1985) or Woodford (1990), among other examples. To the best of my knowledge, I am however the first one to incorporate it in a full fledged business cycle model with capital accumulation in particular. I build on the now very well developed Real Business Cycles literature for solutions techniques and calibration methods.

The rest of the paper proceeds as follows. In Section 1, I develop an overlapping-generations model of the real business cycle, allowing for potential rational bubbles shocks. In Section 2, I discuss more explicitly labor supply elasticities as given by a realistic life-cycle model, to show that the results from the two period model are strengthened. In Section 3, I compute the steady-states and quasi steady-states of the model. I provide evidence for a bubbleless steady-state, a bubbly steady-state, as in Tirole (1985); but also for a continuum of "quasi-steady-states" which I will define below. In section 4, I compute the Impulse Response Functions from rational bubbles shocks and productivity shocks. I discuss the existence of "jobless recoveries": asset prices and consumption can rebound while employment data keeps deteriorating. Finally, section 5 concludes and suggests avenues for future research.

1 The model

The economy consists of an overlapping-generations of a continuum of agents, who work when old and consume when they retire. At each period, the population has a mass 1 of young agents and a mass 1 of old agents. Goods are perishable, but agents can use the capital stock or useless pieces of papers ("bubbles") to save their endowments, as in Tirole (1985). The production side of the economy is standard, similar to Tirole (1985) and Diamond (1965), and also similar to the production side of canonical Real Business Cycle models.

1.1 Production

There is a neoclassical production function taking capital inherited from previous period k_{t-1} and labor n_t as inputs, to produce output y_t with total factor productivity z_t, and constant returns to scale :

$$y_t = z_t k_{t-1}^{\alpha} n_t^{1-\alpha}. \tag{1}$$

Technology is also allowed to depend on time, so that productivity shocks can very well be studied in the model presented below. Firms are competitive, taking wages and rental rate of capital as given, and maximizing their profits. From firms' standard maximization problem, the rental rate and the wages are given by the usual first order conditions:

$$w_t = (1-\alpha) z_t \left(\frac{k_{t-1}}{n_t}\right)^{\alpha} \tag{2}$$

$$r_t = \alpha z_t \left(\frac{n_t}{k_{t-1}}\right)^{1-\alpha} \tag{3}$$

Capital depreciates at a rate δ, but increases through new investment i_t. The capital accumulation equation is therefore :

$$k_t = (1-\delta) k_{t-1} + i_t \tag{4}$$

Let us turn now to the consumers/workers' side of the economy, which differs both from Diamond (1965) and Tirole (1985)'s models and from the Kydland and Prescott (1982) RBC model.

1.2 Labor supply, resource constraints

Agents work n_t when young, consume c_{t+1} when old and have preferences over consumption and leisure given by:

$$U(c_{t+1}, n_t) = \frac{c_{t+1}^{1-\theta}}{1-\theta} - \psi \frac{n_t^{1+\phi}}{1+\phi}.$$

Unlike in Diamond (1965) and Tirole (1985), agents do not supply their labor inelastically: but their labor supply is part of their choice sets, a central theme in the RBC literature. The introduction of a labor supply margin is in no ways new: it is present in Azariadis (1981), Woodford (1990) among many other exa mples. The key to determining the interest rates in this economy is that agents need to store labor income, which they do by buying inherited capital from the old $(1-\delta)k_{t-1}$, investing in new capital i_t and investing in bubbles b_t:

$$b_t + i_t + (1-\delta)k_{t-1} = w_t n_t \tag{5}$$

When old, they consume c_{t+1} given by the realized returns on their investments, both in capital and in the bubble:

$$c_{t+1} = b_{t+1} + (1-\delta + r_{t+1})k_t. \tag{6}$$

The labor supply equation depends in contrast on expected returns from holding both capital and the bubble. Their program is to maximize:

$$\begin{aligned} &\max_{n_t, c_{t+1}} \mathbb{E}U(c_{t+1}, n_t) \\ &\text{s.t.} \quad c_{t+1} = w_t n_t \frac{b_{t+1} + (1-\delta + r_{t+1})k_t}{b_t + k_t} \end{aligned}$$

Under perfect foresight, these returns equal the realized return and expected consumption equals realized consumption:

$$\psi n_t^{\phi} = w_t \mathbb{E}_t^i \left[\frac{b_{t+1} + (1-\delta - r_{t+1})k_t}{b_t + k_t} \right] c_{t+1}^{-\theta} \tag{7}$$

1.3 Discussion

For tractability, a number of simplifying hypotheses have been made, which I here discuss in turn.

Expectations. For simplicity, I work under a common prior assumption. That is, I assume that agents are able to coordinate simultaneously on a Rational Expectations Equilibrium even though there are a number of them with dynamic inefficiency. If it was not the case, then each agent would supply labor differently depending on his relative optimism concerning the future value of the asset. This assumption has been relaxed in Chapter 2, where I have shown in particular that a "rational bubble shock" could be rationalized through a financial liberalization or innovation episode. In that case, heterogeneity could matter in calculating the response of the economy to a shock, and would dramatically impact the response of labor, consumption and investment. Further work should recognize this heterogeneity, as in Krusell and Smith (1998).

Heterogeneity in asset market participation. In the same line of reasoning, it would be interesting to allow for different asset holdings from different part of the population. For example, Figure 1 shows very clearly that housing cycles drive low-frequency consumption movements much more than equity cycles. (note how high dividend-price ratios are not associated to recessions as in the standard Consumption-Asset Pricing Model) This is certainly due to the fact that housing is much more widely held than equity, so that falling housing markets generate substantial disincentives to work, and lower consumption.

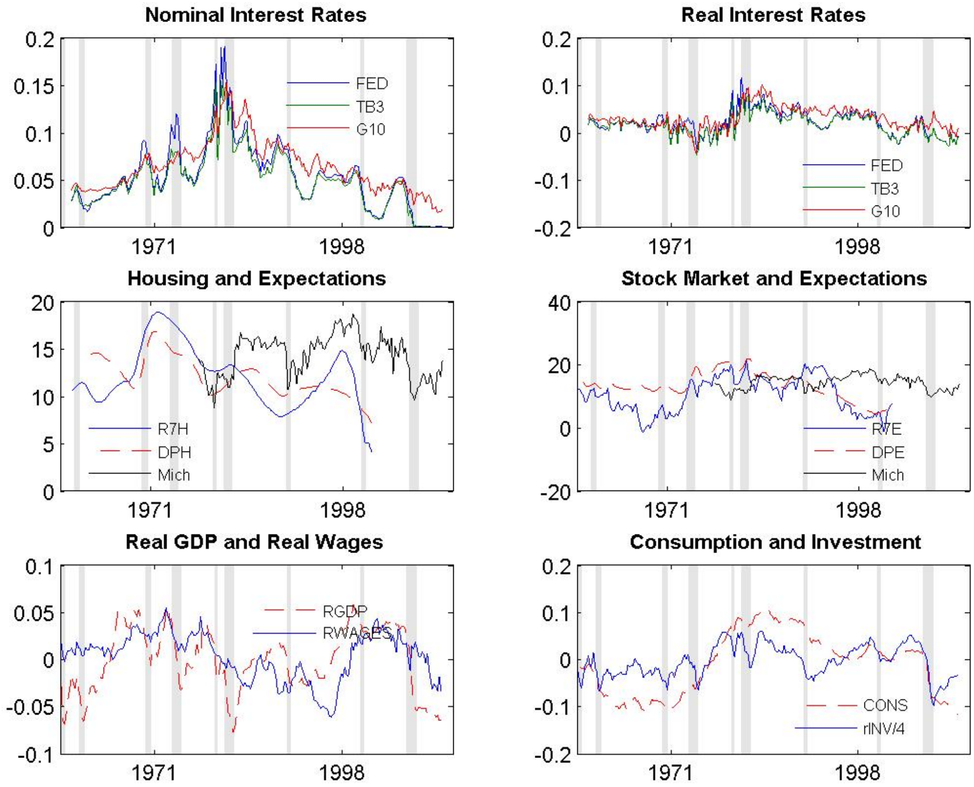
No Euler equation. With the life cycle hypothesis, the Euler equation of an optimizing agent is replaced by a resource constraint when old and young. Because of a complete asynchronicity between production and consumption, agents always are at a corner of their optimization problem. Of course, in reality, this asynchronicity is supplemented by a well-defined life cycle problem, so that some form of the Euler equation does hold. Let me now turn to a more realistic life cycle model with variable labor supply to show that the effects outlined above are still present if not strengthened, before going to the calculations of the steady-state of this model, and looking at shocks in this framework.

2 Labor supply elasticities: A continuous time life cycle model

The OLG 2-period model developed in the sequel of the paper, whereby agents work when young and consume only when old is very stylized, and aimed at capturing the asynchronicity between production and consumption, leading to a demand for stores of value. Such a model however falsely leads one to conclude that balanced growth cum stable number of hours pins down the Intertemporal Elasticity of Substitution (IES), which in turn drives the responses to interest rates. It also falsely leads to a model where the microeconomic Elascticity of Labor Supply (ELS) pins down the responses of hours to returns to work - which, in the OLG model outlined below, are not just the current wage but the wage compounded with interest rate.[5] It is therefore useful to develop a model with several coexisting generations at each point in time. In this section, I therefore develop a continuous time model in the spirit of Tobin (1967), Summers (1981), or Auerbach and Kotlikoff (1987), allowing however for variable labor supply. To the best of my knowledge, I am the first to add such a labor supply choice

[5] In the canonical model with infinite lives, the puzzle is even stronger : macro data would imply labor supply elasticities well above 1, given that wages do not fall much in depressions, while empirical estimates from microeconomic studies point to labor supply elasticities in the $0.1 - 0.5$ range. See Chetty et al. (2011).

Figure 1: US BUSINESS CYCLES FACTS



Note: Data comes from the Federal Reserve Bank of Saint-Louis Database (FRED). FED is the effective Federal Funds Rate, TB3 is the rate on 3 months Treasury-Bills, G10 is the 10-year treasury bond yield. Real interest rates are calculated using the GDP deflator (GDPDEF). R7H is the 7-year ahead real return on housing (the Case-Shiller index is used), DPH is the dividend price ratio on housing, and Mich is the Michigan Index of Consumer Expectations. Similarly, DPE is the 7-year ahead real return on a stock index, and DPE is the dividend price ratio on stocks. RGDP is real gdp, RWAGES real wages, CONS is personal consumption expenditures, rINV is real gross private domestic investment. RGDP, RWAGES, RCONS, rINV are log deviations from a trend calculated using a HP filter of very high parameter (10^7). See Figures 6, 7, 8, 9, 10, 11 in the Appendix to see this data in more detail.

in a more realistic overlapping generations model with continuous time ; I think it can help shed a new light on the micro/macro labor supply elasticity conundrum.

2.1 Set-up

This section provides an alternative to the two-period formulation of the OLG model. This model takes into account the fact that labor supply is also smoothed over time taking into account the relative potency of economic growth (raising wages), interest rates (important if wages are to be consumed later), and impatience.[6] Moreover, rising interest rates, which are

[6]Note that unlike in the infinite horizon models, where impatience is required for the agents' optimization problem to be well-defined, here impatience could be set to 0 or even negative if agents enjoy consumption later in life.

homothetic to a rise in wages in the 2-period formulation (thus having an effect only through the ELS), have less intuitive effects in this continuous time, more realistic description. I will turn back to the intuition after presenting the results.

On the consumer/worker side, individuals born at time s (which I later call generation s) both choose a consumption plan $\{C_{st}\}_{t=0}^{T}$ and a labor supply plan $\{L_{st}\}_{t=0}^{T'}$ to maximize an intertemporal utility function, subject to a lifetime budget constraint:

$$\begin{aligned} \max_{\{C_{st},L_{st}\}} \int_0^T U(C_{st})e^{-\delta t}dt - \int_0^{T'} V(L_{st})e^{-\delta t}dt \\ \text{s.t.} \quad \int_0^T C_{st}e^{-rt}dt \leq \int_0^{T'} w_{t+s}L_{st}e^{-rt}, \end{aligned}$$

where t denotes time since an agent from generation s was born, T is the certain time of death[7], T' the certain age for retirement[8], w_t is the wage rate assumed to grow at an exogenous rate of technological change g, L_{st} is agents' from generation s labor supply at date $s+t$, δ is a discount factor. In order to make the problem tractable, and following the RBC literature, I assume a constant elasticity for both utility for consumption and disutility of labor. For simplicity, I do not distinguish between the power function with a 0 exponent and the logarithm function in the following, I however have checked that the results carry over to this case independently. Consumers from generation s therefore solve the following program:

$$\begin{aligned} \max_{\{C_{st},L_{st}\}} \int_0^T C_{st}^{\sigma}e^{-\delta t}dt - \int_0^{T'} L_{st}^{\epsilon}e^{-\delta t}dt \\ \text{s.t.} \quad \int_0^T C_{st}e^{-rt}dt \leq w_0\int_0^{T'} L_{st}e^{-rt+g(s+t)}, \end{aligned}$$

Intertemporal maximization leads to (see Appendix 1.1):

$$\begin{aligned} \forall t \in [0,T], \quad C_{st} = C_{s0}e^{\frac{r-\delta}{1-\sigma}t} \\ \forall t \in [0,T'], \quad L_{st} = L_{s0}e^{\frac{r-\delta-g}{1-\epsilon}t}. \end{aligned}$$

Note that the labor decision profile is increasing over time when interest rates are relatively high (dynamic efficiency - $r > \delta + g$). In contrast, agents tend to prefer to work late in life when $r < \delta + g$, because the store of value technology is not very efficient. This is another channel in this model through which declining interest rates in bubbleless times makes labor supply decline.

[7] See Merton (1969) for a way to generalize this under uncertainty. It does not change the results.

[8] Work is assumed to be impossible after this date for exogenous technological reasons. The disutility of work could be assumed to be infinite for $t \geq T'$.

2.2 Balanced growth restriction

This section shows that the well-known result in growth theory remains here valid. In order to justify the non-explosiveness (and not vanishing character) or hours worked across time, one has to assume $\sigma = 0$, or a unit elasticity of intertemporal substitution with offsetting substitution and income effects.

Replacing the life-cycle profile of consumption and labor in generation s agents' utility , one gets:

$$U_s\left(\{C_{st}\}_{t=0}^{T}, \{L_{st}\}_{t=0}^{T'}\right) = C_{s0}^{\sigma}\int_0^T e^{\left[\frac{r-\delta}{1-\sigma}\sigma-\delta\right]t}dt - L_{s0}^{\epsilon}\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}\epsilon-\delta\right]t}dt.$$

The budget constraint similarly rewrites:

$$C_{s0}\int_0^T e^{\left[\frac{r-\delta}{1-\sigma}-r\right]t}dt \leq e^{gs}w_0 L_{s0}\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}-r+g\right]t}dt$$

Maximizing with respect to initial values for consumption and leisure therefore leads to (for $\sigma \neq 0$):

$$\frac{\epsilon L_{s0}^{\epsilon-1}}{\sigma C_{s0}^{\sigma-1}}\frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}\epsilon-\delta\right]t}dt}{\int_0^{T} e^{\left[\frac{r-\delta}{1-\sigma}\sigma-\delta\right]t}dt} = e^{gs}w_0\frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}-r+g\right]t}dt}{\int_0^{T} e^{\left[\frac{r-\delta}{1-\sigma}-r\right]t}dt}.$$

For $\sigma = 0$ (logarithmic utility),

$$\epsilon L_{s0}^{\epsilon-1}C_{s0}\frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}\epsilon-\delta\right]t}dt}{\int_0^{T} e^{\left[\frac{r-\delta}{1-\sigma}\sigma-\delta\right]t}dt} = e^{gs}w_0\frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}-r+g\right]t}dt}{\int_0^{T} e^{\left[\frac{r-\delta}{1-\sigma}-r\right]t}dt}.$$

From this equation it is apparent that only $\sigma = 0$ allows balanced growth (since both wages and consumption grow at g). What is very interesting in this model however is that labor supply still responses to income after compounded interest.

2.3 Average labor supply

Because of the previous remark, let me operate now under the assumption of unit elasticity of substitution $\sigma = 0$.[9]

Labor supply is therefore constant across cohorts $L_{s0} = L_0$. Using balanced growth that is $C_{s0} = e^{gs}C_0$. More precisely, it is given by (again recall that the Elasticity of Labor Supply is $1/(\epsilon - 1)$ in this model) - See Appendix 1.1 for a detail for these simplications :

[9]One could do without this assumption and use $\sigma \neq 0$ throughout, however making such an assumption makes one's life considerably harder.

$$L_{s0} = L_0 = \left[\frac{w_0}{\epsilon C_0}\right]^{\frac{1}{\epsilon-1}}.$$

In this model, agents supply their labor heterogenously, not just because some are working and other are retired, but also because agents are not at the same point in their labor decision profiles. However, one can calculate average labor supply at time t $\bar{L_t}$ in a tractable way:

$$\int_{t-T}^{t} \bar{L_t} e^{n(s-t)} ds = \int_{t-T}^{t} L_{s0} e^{\frac{r-g-\delta}{1-\epsilon}(t-s)} e^{n(s-t)} ds.$$

After some simple algebra, this gives:

$$\bar{L_t} = \frac{e^{\left(\frac{r-g-\delta}{1-\epsilon}-n\right)T} - 1}{\frac{r-g-\delta}{1-\epsilon} - n} \frac{n}{1-e^{-nT}} L_0 = \frac{e^{\left(\frac{r-g-\delta}{1-\epsilon}-n\right)T} - 1}{\frac{r-g-\delta}{1-\epsilon} - n} \frac{n}{1-e^{-nT}} \left[\frac{w_0}{\epsilon C_0}\right]^{\frac{1}{\epsilon-1}}.$$

Note that labor supply is here independent of time, I denote it by $\bar{L_0}$ in the following. Moreover, I denote by ϵ_L the Elasticity of Labor Supply equal to $1/(\epsilon - 1)$. Then the interest elasticity of Labor Supply is given by:

$$\frac{1}{\bar{L_0}} \frac{\partial \bar{L_0}}{\partial r} = \frac{1}{\delta + g - r} \left[1 - \frac{\epsilon_L(\delta + g - r)T}{e^{\epsilon_L(\delta+g-r)T} - 1}\right].$$

To get an intuition for this formula, let me calibrate it as $\epsilon = 0.1$, in the ballpark of microeconomic estimates for labor supply elasticity, $T = 60$ for working and retirement life (so that ϵT is of the order of 6). Let me assume that $\delta + g - r$ is "small" (somewhat arbitrarily, δ can be calibrated between 0 and 0.03, growth at 0.02, and interest rate at a similar magnitude, so that $\delta + g - r$ is "small" indeed). Then approximating the previous equation in the vicinity of 0 yields:

$$\frac{1}{\bar{L_0}} \frac{\partial \bar{L_0}}{\partial r} = \frac{1}{\delta + g - r} \left[1 - \frac{\epsilon_L T(\delta + g - r)}{\epsilon_L T(\delta + g - r) + \frac{(\epsilon_L T)^2}{2}(\delta + g - r)^2 + \circ((\delta + g - r)^2)}\right] = \frac{\epsilon_L T}{2} + \circ(1).$$

In the very simple case presented here, the elasticity of labor supply with respect to the interest rate can therefore be very high, even if ϵ_T is low.

3 Steady-states

3.1 Calculation for a hypothetical r^* steady-state

Unlike Tirole (1985), I will in this section allow the bubble to be a constant fraction of the endowment (growing at g or 0 in case of no growth), with agents expecting that it will grow at r. Therefore I will be able to have $b^* \neq 0$ as well as $1 + r^* - \delta \neq 1$.Because of no arbitrage between the bubbles' return and the return on capital, there is a free parameter in the economy. I take this parameter as r^*. Note that there is a one-to-on mapping between bubble per hours of work b^*/n^* and r^*, from equations (1), (2) and (3):

$$\frac{b^*}{n^*} = (1-\alpha)z^* \left(\frac{k^*}{n^*}\right)^{\alpha} - \frac{k^*}{n^*}$$

This leads to (see Appendix 1.2 for details) :

$$\frac{b^*}{n^*} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(z^*)^{\frac{1}{1-\alpha}}\left[(r^*)^{-\frac{\alpha}{1-\alpha}} - \frac{\alpha}{1-\alpha}(r^*)^{-\frac{1}{1-\alpha}}\right].$$

When the capital stock is above the Golden Rule level of capital accumulation, this function defines a bijective increasing mapping between the level of the equilibrium bubble per hours worked and the interest rate (refer to Appendix 1.2 for a proof).

The steady-state can then be computed as a function of this equilibrium interest rate r^*:

$$\frac{k^*}{n^*} = \left(\frac{\alpha z^*}{r^*}\right)^{\frac{1}{1-\alpha}} \qquad \text{(Firm choice of } k\text{)}$$

$$w^* = z^*(1-\alpha)\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{(Firm choice of } l\text{)}$$

$$\frac{y^*}{n^*} = z^*\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{(Production Function)}$$

$$\frac{b^*}{n^*} = z^*(1-\alpha)\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha z^*}{r^*}\right)^{\frac{1}{1-\alpha}} \qquad \text{(Unproductive Savings)}$$

$$\frac{c^*}{n^*} = (1-\alpha)z^*\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}}(1-\delta+r^*) \qquad \text{(Realized Consumption)}$$

$$n^* = \psi^{-\frac{1}{\phi+\theta}}[z^*(1-\alpha)]^{\frac{1-\theta}{\phi+\theta}}\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha(1-\theta)}{(1-\alpha)(\phi+\theta)}}(1-\delta+r^*)^{\frac{1-\theta}{\phi+\theta}} \qquad \text{(Labor Supply).}$$

3.2 Competitive bubbleless equilibrium

In the bubbleless equilibrium, per hour bubble is constant and equal to 0 ($b^* = 0$). Therefore, the previous equations lead to (see Appendix 1.6):

$$\frac{k^*}{n^*} = [(1-\alpha)z^*]^{\frac{1}{1-\alpha}} \qquad w^* = [(1-\alpha)z^*]^{\frac{1}{1-\alpha}}$$

$$\frac{y^*}{n^*} = (1-\alpha)^{\frac{\alpha}{1-\alpha}}(z^*)^{\frac{1}{1-\alpha}}$$

$$\frac{c^*}{n^*} = [1-\delta(1-\alpha)](z^*)^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{\alpha}{1-\alpha}}$$

$$n^* = \psi^{-\frac{1}{\phi+\theta}}[(z^*)(1-\alpha)^{\alpha}]^{\frac{1-\theta}{(1-\alpha)(\phi+\theta)}}[1-\delta(1-\alpha)]^{\frac{1-\theta}{\phi+\theta}}$$

These 5 non-linear equations define an equilibrium $(k^*, n^*, w^*, y^*, c^*)$ as a function of technology z^*. Let us turn now to the case where the rational bubble is a constant fraction of GDP, initially at a level such that the interest rate is at its Golden-Rule level (as in Tirole (1985)).

3.3 Competitive bubbly equilibrium

In the bubbly steady-state, the size of the bubble is determined by the arbitrage condition given by $1 + r^* - \delta = 1$, which makes the size of the bubble constant. This is the knife edge case where the bubble does not vanishes asymptotically or explode and become greater than the size of the endowment, as in Tirole (1985). One is now lead to the following equations:

$$\frac{k^*}{n^*} = (z^*)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}} \qquad w^* = (z^*)^{\frac{1}{1-\alpha}} (1-\alpha) \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\frac{y^*}{n^*} = (z^*)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\frac{b^*}{n^*} = (z^*)^{\frac{1}{1-\alpha}} \left[(1-\alpha)\left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right]$$
$$\frac{c^*}{n^*} = (z^*)^{\frac{1}{1-\alpha}} (1-\alpha) \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$
$$n^* = \psi^{-\frac{1}{\phi+\theta}} (z^*)^{\frac{1}{1-\alpha}\frac{1-\theta}{\phi+\theta}} (1-\alpha)^{\frac{1-\theta}{\phi+\theta}} \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\phi+\theta}}.$$

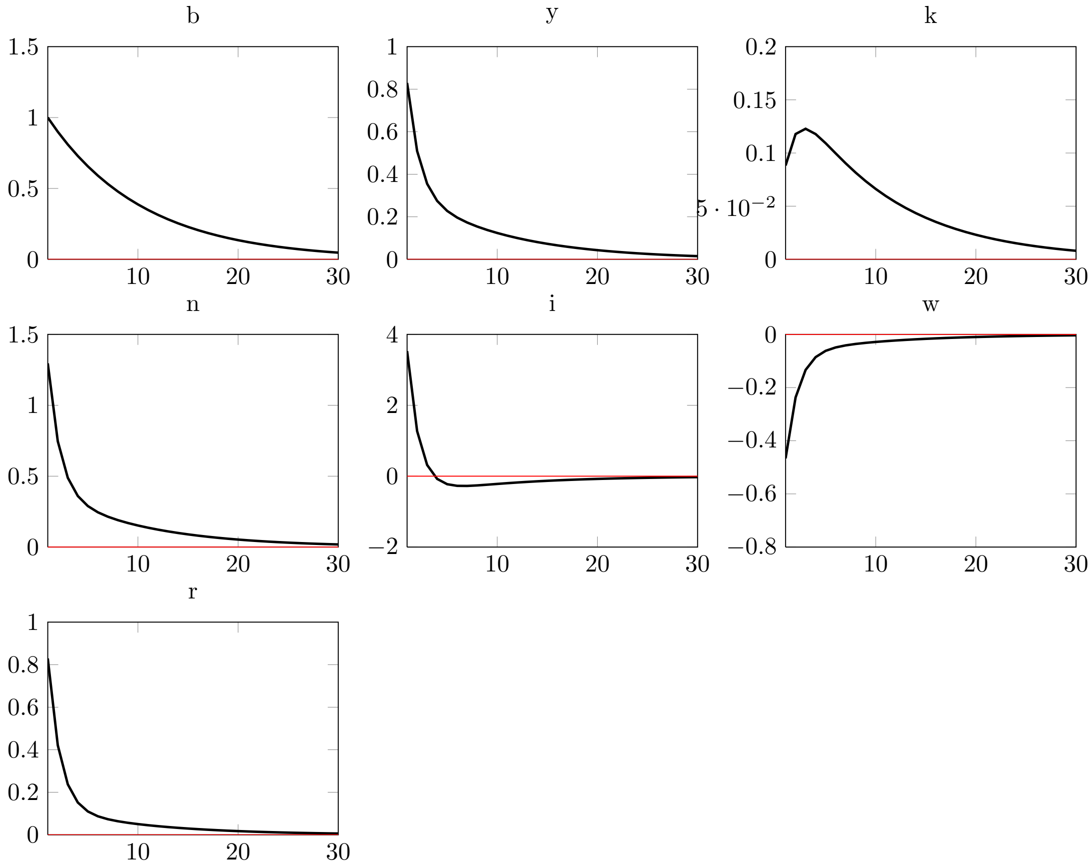
These 6 non-linear equations define an equilibrium $(k^*, n^*, w^*, y^*, c^*, b^*)$ as a function of technology z^*. Note that depending on calibrated parameters, the bubbly equilibrium can have higher employment, capital, investment and consumption than the bubbleless equilibrium. The intuition behind this result is that if the bubble increases the efficiency with which resources are transferred through time: first through its direct effect of transferring resources with an interest rate equal to the rate of growth, and second by increasing efficiency of the remaining capital stock whose return increases. Therefore, it can become more valuable for agents to work because of these higher returns, because the efficiency of the store of value technology has increased.

4 Shocks

The model is calibrated as the standard Real Business Cycle models. Note that as was made clear in section 2, one does not need a high elasticity of labor supply to match the data because the elasticity of labor to the interest rate can be much higher than the microeconomic elasticity ($\epsilon_L T$ in the case considered above). Moreover, it is possible to choose an elasticity of intertemporal substitution equal to 1 to match balanced growth, while maintaining high effects on labor supply. The impulse response from a rational bubbles shock are displayed on Figure 2 for the case of initially high interest rates, and on Figure 3 for the case where interest rates are low initially. Note that this last case shows that there exists a "liquidity trap" when the economy is stuck with too low interest rates.

Note that in the model, and because of the shape of the labor supply function, rebounds in the stock market with increasing consumption due to the rebound of rational bubbles

Figure 2: IMPULSE RESPONSE FUNCTIONS FROM A "BUBBLE SHOCK": HIGH INTEREST RATES



Note: Impulse response functions from a "bubble shock" when the interest rate is initially high. There is positive comovement between hours,. Note that productivity numbers can also increase in particular if workers provide higher work effort.

can be accompanied by initially decreasing levels of employment. Even more than "jobless recoveries", the model rationalizes "unemployment-increasing recoveries". (see Figure 4)

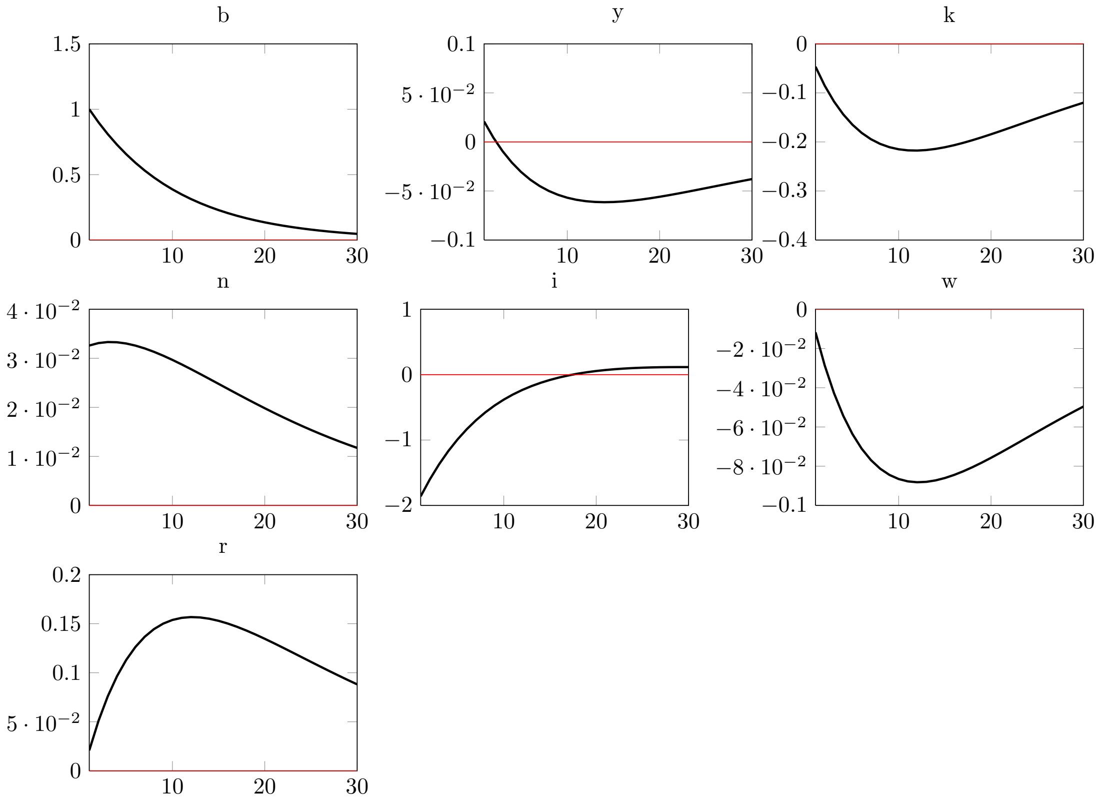
5 Conclusion

This paper has shown that rational bubbles are compatible with a procyclical behavior of investment, consumption, and hours, even in a perfectly neoclassical setting (without information asymmetries, as in Farhi and Tirole (2011)).[10]

Of course, rational bubbles can only appear in a model where agents have finite lives, instead of being infinitely lived. In fact, using a life-cycle model without rational bubbles shocks has its own merits as I have shown, and can help explain many puzzles in macroeconomics and in the study of economic fluctuations. A first puzzle it helps solve is why real interest rates move so much: to the extent that they are determined in the long run by the rate of

[10]This is not to deny the importance of information asymmetries. Adding those effects would only reinforce the view that bubbles lead to investment booms.

Figure 3: Impulse response functions from a "bubble shock": low interest rates

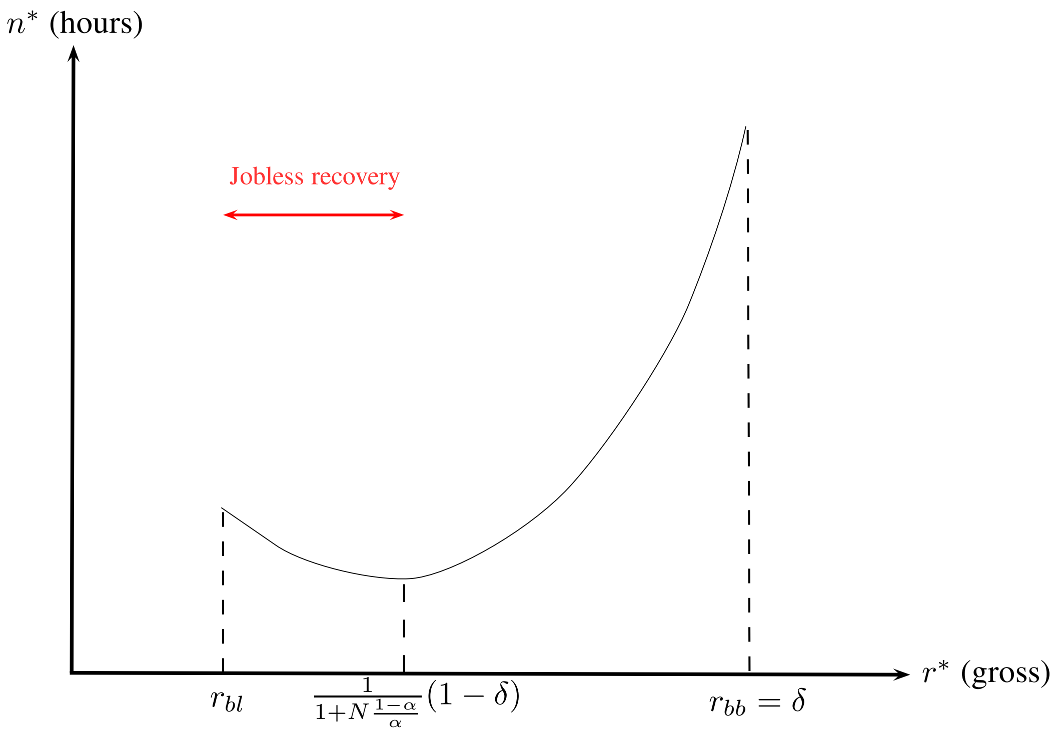


Note: Impulse response functions from a "bubble shock" when the interest rate is initially low ("crisis mode" or "liquidity trap"). Note that a bubble shock in that case leads to lower capital accumulation.

time preference as in Fisher (1930), their long run movements should reflect preference shocks. There is no need for such shocks in the model presented here: some changes in demography, or endowments between the different moments in life (or the rise and fall of rational bubbles) can generate significant movements in real interest rates. A second interesting result from life-cycle models is that elasticities of labor supply can be very high macroeconomically in response to changes in interest rates, while microeconomic data suggests that they are relatively small with respect to identified changes in contemporaneous tax rates. (Chetty et al. (2011)) Once again, such a claim deserves more rigorous calibration and empirical analysis.

In many respects, this paper is a very first step into the study of rational bubbles driven business cycles. The paper should be extended in many important ways. First, I have assumed a closed economy which was hence not interacting with the rest of the world. A natural extension involves developing an International Business Cycles version of the model. Qualitatively, the model helps resolve many of the puzzles outlined in Backus et al. (1992), and even most of the six puzzles detailed in Obstfeld and Rogoff (2001). For example, equity home bias is a natural byproduct of dynamic inefficiency and even infinitesimally small costs of trading assets across borders (in any case, taxes are such costs). Whether it can solve them quantitatively is a question for future research.

Figure 4: Jobless recovery



Of course, this theory of the business cycle raises a number of important issues. It suggests that the familiar New-Keynesian proposition that increases in money supply do not have any effect in the long run, but only affect the short-run can be misguided. In particular, although it was not modelled here because the model was very crude, monetary policy can have some direct effect on the business cycle through a credit channel, which raises asset prices and allows for higher consumption. If this is true, then attempts to identify the effect of monetary shocks through structural Vector Auto-Regressions and long-run restrictions will perhaps not prove very fruitful: monetary policy perhaps has some effects for the longer run.

A final remark is that viewing the business cycle as a succession of rational bubbles shocks, even though non-conventional, leads to embrace many Keynesian recommendations, while helping to qualify some of them. To the best of my knowledge, this connection between Allais (1947) and Samuelson (1958)'s theory of general equilibrium with Keynes (1936)'s theory of non-clearing markets was first remarked by Geanakoplos (2008), who underlined some similarities between the two: the multiplicity of equilibria and hence the role for animal spirits, the determination of the rate of interest independently of the rate of time preference, etc. This theory of rational bubbles driven business cycles suggests that monetary policy can have effects beyond those connected to money as a unit of account, but also through a credit channel allowing lower interest rates (or lower quality collateral) when lending requirements are relaxed. It similarly suggests a role for public debt when asset prices are low and investors are caught in a bad employment trap, and monetary policy is no longer powerful enough to help optimists bid up asset prices.

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1 Omitted Proofs

1.1 Continuous time model

Proof. The problem to solve is standard:

$$\begin{aligned}\max_{\{C_{st},L_{st}\}} \int_0^T C_{st}^{\sigma} e^{-\delta t} dt - \int_0^{T'} L_{st}^{\epsilon} e^{-\delta t} dt \\ \text{s.t.} \int_0^T C_{st} e^{-rt} dt \leq w_0 \int_0^{T'} L_{st} e^{-rt+g(s+t)}.\end{aligned}$$

Setting up the Lagrangian of the problem with one multiplier λ on the budget constraint, one has to solve:

$$\max_{\{C_{st},L_{st}\}} \int_0^T C_{st}^{\sigma} e^{-\delta t} dt - \int_0^{T'} L_{st}^{\epsilon} e^{-\delta t} dt + \lambda \left(w_0 \int_0^{T'} L_{st} e^{-rt+g(s+t)} - \int_0^T C_{st} e^{-rt} dt \right).$$

Pick t, and write that the ratio of marginal flow utilities in an interval dt around 0 and t are equal to consumption prices of consuming at the initial period and at t. Similarly for labor, for $t \in [0, T']$. This leads to:

$$\frac{\sigma C_{st}^{\sigma-1} e^{-\delta t} dt}{\sigma C_{s0}^{\sigma-1} dt} = e^{-rt} \qquad \frac{\epsilon L_{st}^{\epsilon-1} e^{-\delta t} dt}{\epsilon L_{s0}^{\epsilon-1} dt} = \frac{w_0 e^{-rt+g(s+t)}}{w_0 e^{gs}}.$$

Rearranging:

$$\begin{aligned}\forall t \in [0,T], \quad C_{st} = C_{s0} e^{\frac{r-\delta}{1-\sigma} t} \\ \forall t \in [0,T'], \quad L_{st} = L_{s0} e^{\frac{r-\delta-g}{1-\epsilon} t}.\end{aligned}$$

□

Proof. The expressions for labor supply are actually a lot simpler than they seem in the main text. For example, in expression:

$$\epsilon L_{s0}^{\epsilon-1} C_{s0} \frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}\epsilon-\delta\right]t} dt}{\int_0^T e^{\left[\frac{r-\delta}{1-\sigma}\sigma-\delta\right]t} dt} = e^{gs} w_0 \frac{\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}-r+g\right]t} dt}{\int_0^T e^{\left[\frac{r-\delta}{1-\sigma}-r\right]t} dt}.$$

it is clear that the two denominators are equal:

$$\int_0^T e^{\left[\frac{r-\delta}{1-\sigma}-r\right]t} dt = \int_0^T e^{\left[\frac{r-\delta}{1-\sigma}\sigma-\delta\right]t} dt.$$

The two numerators are equal as well:

$$\int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}\epsilon-\delta\right]t}dt = \int_0^{T'} e^{\left[\frac{r-\delta-g}{1-\epsilon}-r+g\right]t}dt.$$

Therefore,

$$\begin{aligned} \epsilon L_{s0}^{\epsilon-1}C_{s0} &= e^{gs}w_0 \quad \text{for} \quad \sigma = 0, \\ \epsilon L_{s0}^{\epsilon-1}C_{s0}^{1-\sigma} &= \sigma e^{gs}w_0 \quad \text{for} \quad \sigma \neq 0. \end{aligned}$$

□

1.2 Steady-state: mapping $r^* \mapsto b^*$

The mapping between bubble per hour and capital per hour is given by:

$$\frac{b^*}{n^*} = (1-\alpha)z^*\left(\frac{k^*}{n^*}\right)^\alpha - \frac{k^*}{n^*}$$

b^*/n^* is a convex function of k^*/n^*, decreasing for all $k^*/n^* > k^{GR}/n^{GR}$ (the golden rule level of capital per hour maximizes it by definition in this economy with $g = 0$). Therefore the interest rate r^*, a decreasing monotone function of capital per hour, is strictly increasing in b^*/n^*. The map $[-\infty;\delta] \to \mathbb{R}$ given by $x \mapsto (1-\alpha)z^*x^\alpha - x$ is therefore bijective.

1.3 Steady-state: calculations

As explained in the main text, there is an infinity of competitive equilibria depending on the value of the interest rate r^*. Equation (3) gives:

$$r^* = \alpha z^*\left(\frac{n^*}{k^*}\right)^{1-\alpha} \quad \Rightarrow \quad \left(\frac{k^*}{n^*}\right)^{1-\alpha} = \frac{\alpha z^*}{r^*} \quad \Rightarrow \quad \frac{k^*}{n^*} = \left(\frac{\alpha z^*}{r^*}\right)^{\frac{1}{1-\alpha}}.$$

From capital-per-hour, one gets the wage with equation (2):

$$w^* = z^*(1-\alpha)\left(\frac{k^*}{n^*}\right)^\alpha \quad \Rightarrow \quad w^* = z^*(1-\alpha)\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}}$$

Plugging equation (4) into equation (1) yields, at the steady state:

$$\frac{b^*}{n^*} + \frac{k^*}{n^*} = w^* \quad \Rightarrow \quad \frac{b^*}{n^*} = z^*(1-\alpha)\left(\frac{\alpha z^*}{r^*}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha z^*}{r^*}\right)^{\frac{1}{1-\alpha}}.$$

1.4 Study of the function $r^* \mapsto (1-\delta+r^*)^a(r^*)^{-b}$

The map $\mathbb{R}_+^* \to \mathbb{R}$ defined by $r^* \mapsto (1-\delta+r^*)^a(r^*)^{-b}$, with $a \geq 0, b \geq 0$ comes back recurrently in the study of hours worked or capital so it is useful to study it systematically.

Defining:

$$\forall r \in \mathbb{R}_+^*, \quad f(r) = \frac{1}{r^b}(1-\delta+r)^a,$$

gives:

$$\begin{aligned} f'(r) &= \frac{a(1-\delta+r)^{a-1}r^b - br^{b-1}(1-\delta+r)^a}{r^{2b}} \\ &= \frac{r^{b-1}(1-\delta+r)^{a-1}}{r^{2b}}\left[(a-b)r-(1-\delta)b\right]. \end{aligned}$$

With $a \leq b$, the function is decreasing on $\mathbb{R}_+^*$. With $a > b$, the function is decreasing on $\left]0, \frac{b(1-\delta)}{a-b}\right]$ and increasing on $\left[\frac{b(1-\delta)}{a-b}, +\infty\right[$.

1.5 Log-linearization of the model

Log linearization of the production (1) is straightforward:

$$y_t = z_t k_{t-1}^{\alpha} n_t^{1-\alpha} \quad \Rightarrow \quad \widetilde{y_t} = \widetilde{z_t} + \alpha\widetilde{k_{t-1}} + (1-\alpha)\widetilde{n_t}.$$

The wage and the rental rate from firms' optimization similarly give:

$$\begin{aligned} w_t &= (1-\alpha)z_t\left(\frac{k_{t-1}}{n_t}\right)^{\alpha} \quad &\Rightarrow \quad \widetilde{w_t} &= \widetilde{z_t} + \alpha(\widetilde{k_{t-1}} - \widetilde{n_t}) \\ r_t &= \alpha z_t\left(\frac{n_t}{k_{t-1}}\right)^{1-\alpha} \quad &\Rightarrow \quad \widetilde{r_t} &= \widetilde{z_t} + (1-\alpha)(\widetilde{n_t} - \widetilde{k_{t-1}}). \end{aligned}$$

The investment equation (1) (young agents invest in real assets and the bubble) writes:

$$b_t + i_t + (1-\delta)k_{t-1} = w_t n_t \quad \Rightarrow \quad \frac{b^*}{w^*n^*}\widetilde{b_t} + \frac{i^*}{w^*n^*}\widetilde{i_t} + \frac{k^*}{w^*n^*}(1-\delta)\widetilde{k_{t-1}} = \widetilde{w_t} + \widetilde{n_t}.$$

The consumption equation (6) (old agents consume their returns from real asset and the bubble) writes:

$$\begin{aligned} c_{t+1} &= b_{t+1} + (1-\delta+r_{t+1})k_t \\ \Rightarrow \quad \widetilde{c_{t+1}} &= \frac{b^*}{b^*+(1-\delta+r^*)k^*}\widetilde{b_t} + \frac{1-\delta+r^*}{b^*+(1-\delta+r^*)k^*}\widetilde{k_t} + \frac{1-\delta+r^*}{b^*+(1-\delta+r^*)k^*}\frac{r^*}{1-\delta+r^*}\widetilde{r_{t+1}} \\ \Rightarrow \quad \widetilde{c_{t+1}} &= \frac{b^*}{b^*+(1-\delta+r^*)k^*}\widetilde{b_t} + \frac{1-\delta+r^*}{b^*+(1-\delta+r^*)k^*}\widetilde{k_t} + \frac{r^*}{b^*+(1-\delta+r^*)k^*}\widetilde{r_{t+1}}. \end{aligned}$$

The capital accumulation equation (4) writes:

$$k_t = (1-\delta)k_{t-1} + i_t \quad \Rightarrow \quad \widetilde{k_t} = \frac{(1-\delta)k^*}{(1-\delta)k^*+i^*}\widetilde{k_{t-1}} + \frac{i^*}{(1-\delta)k^*+i^*}\widetilde{i_t}.$$

Assuming that all agents have the same expectations about returns, and that they think the return on the bubble will be the same:

$$\phi n_t^{\phi} = w_t \mathbb{E}_t^i \left[\frac{b_{t+1} + (1 - \delta - r_{t+1})k_t}{b_t + k_t} \right] c_{t+1}^{-\theta} \quad \Rightarrow \quad \widetilde{n_t} = \frac{1}{\phi}\widetilde{w_t} + \frac{r^*}{\phi(1 - \delta + r^*)}\mathbb{E}[\widetilde{r_{t+1}}] - \frac{\theta}{\phi}\widetilde{c_{t+1}}.$$

1.6 Bubbleless steady-state

At the bubbleless steady-state, $b^* = 0$. This allows to calculate the steady state interest rate:

$$\frac{k^*}{n^*} = \left[(1 - \alpha)z^*\right]^{\frac{1}{1-\alpha}}.$$

2 Simulations

2.1 Details of Simulations

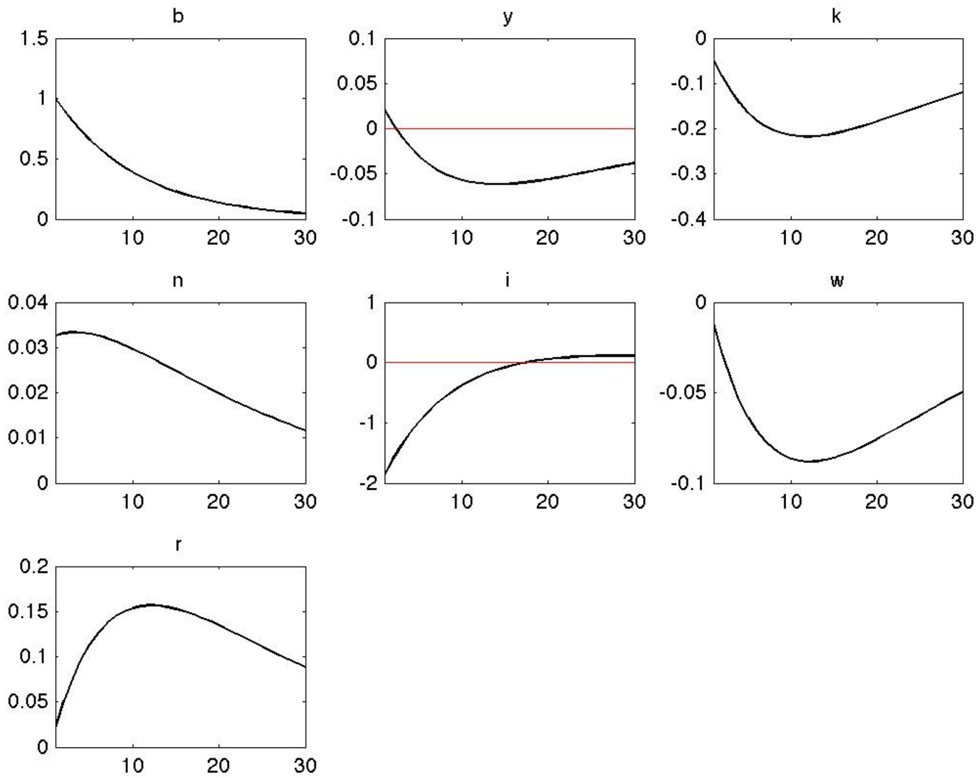
An important difference between this model and the plain RBC model is that the interest rate is not determined by a transversality condition on consumption giving $\beta(1 - \delta + r_{t+1})$, or the Fisher (1930) theory of interest rates depending on consumers' impatience:

$$r^* = \delta + \frac{1}{\beta} - 1$$

In contrast, consumption here is determined by available resources when old, itself depending on investment in real assets and bubble assets. Therefore, when using consumption as a variable in Dynare, it is important to recognize that it is no longer a variable looking forward to infinity as in infinite horizon models.

2.2 Other simulations

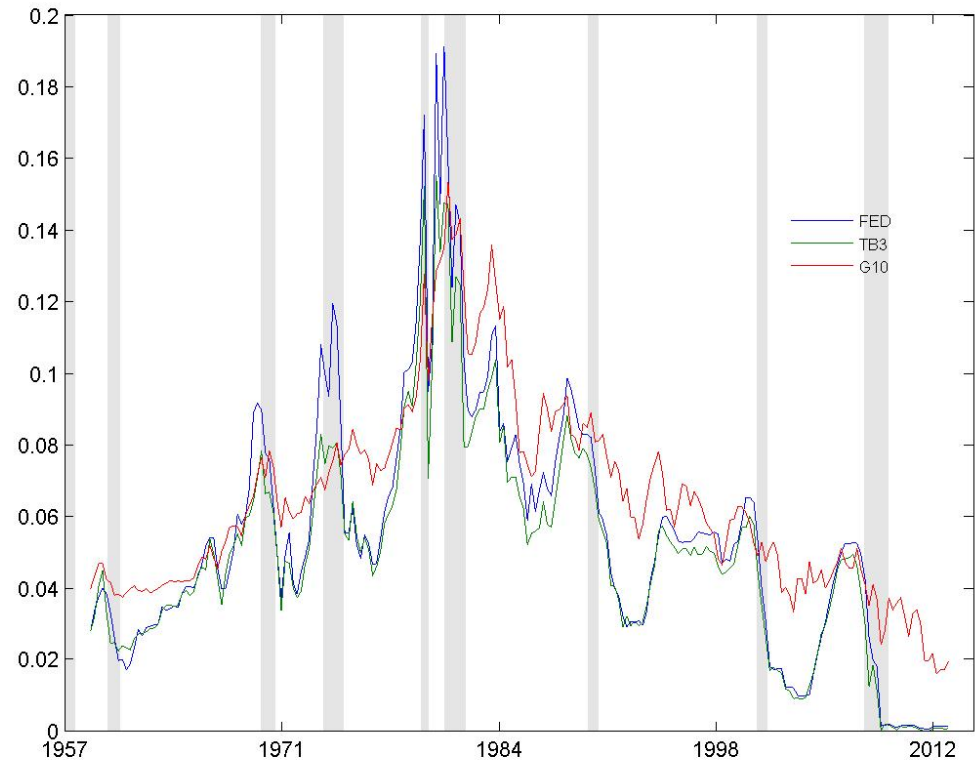
Figure 5: Impulse response functions from a "productivity shock": low interest rates



Note: Impulse response functions from a productivity shock.

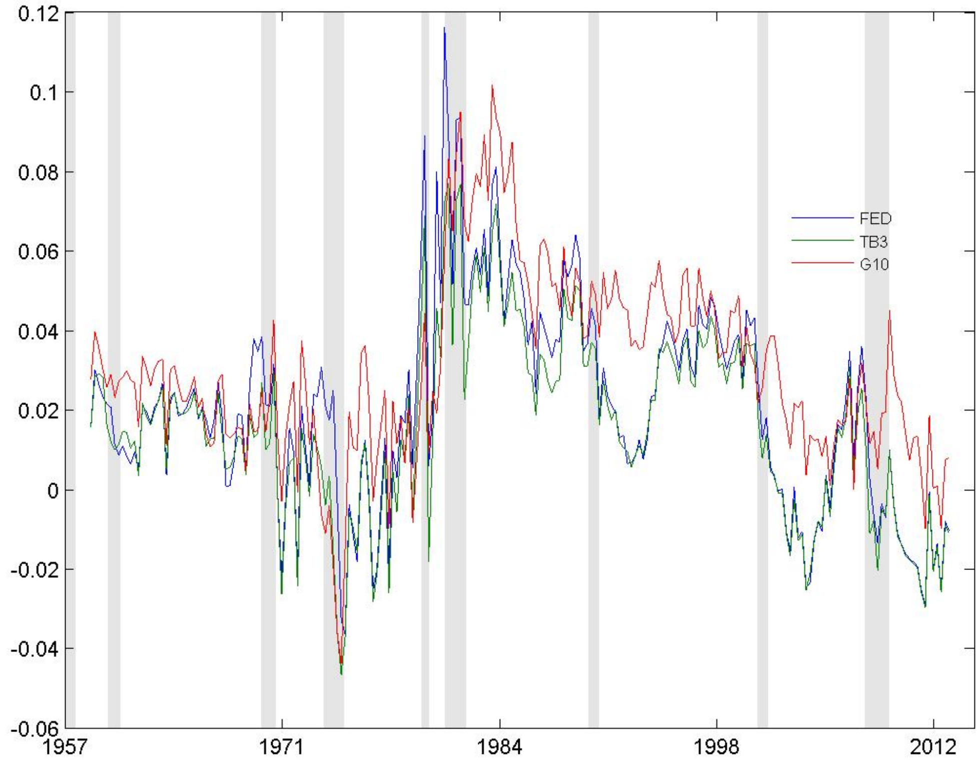
3 Figures

Figure 6: Nominal Interest Rates



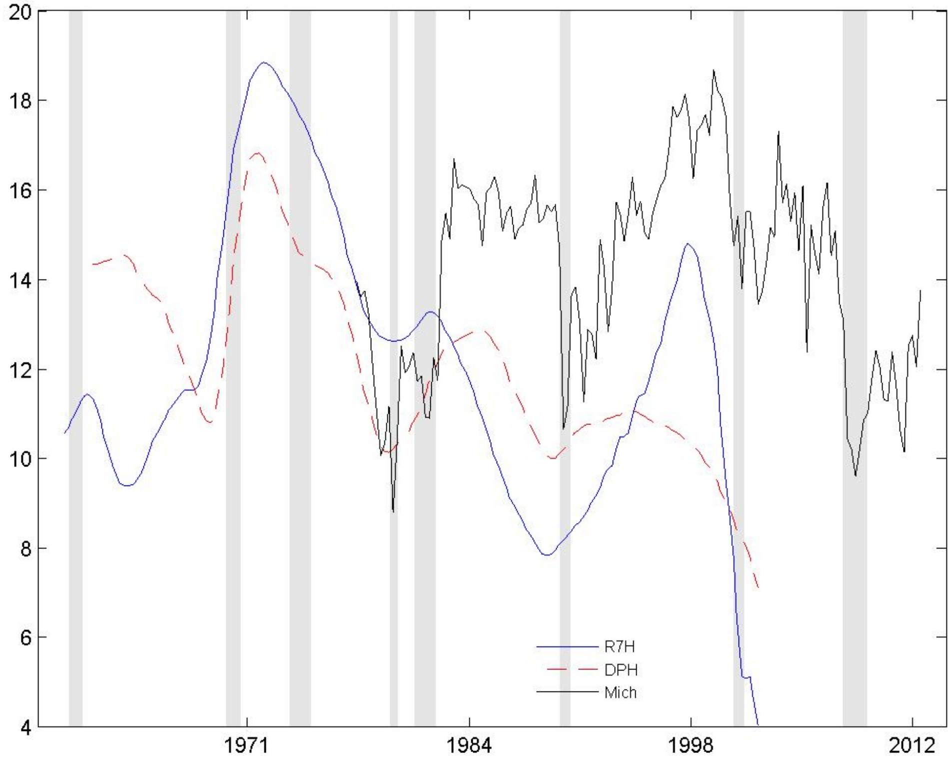
Note: FED is the effective Federal Funds Rate, TB3 is the rate on 3 months Treasury-Bills, G10 is the 10-year treasury bond yield.

Figure 7: Real Interest Rates



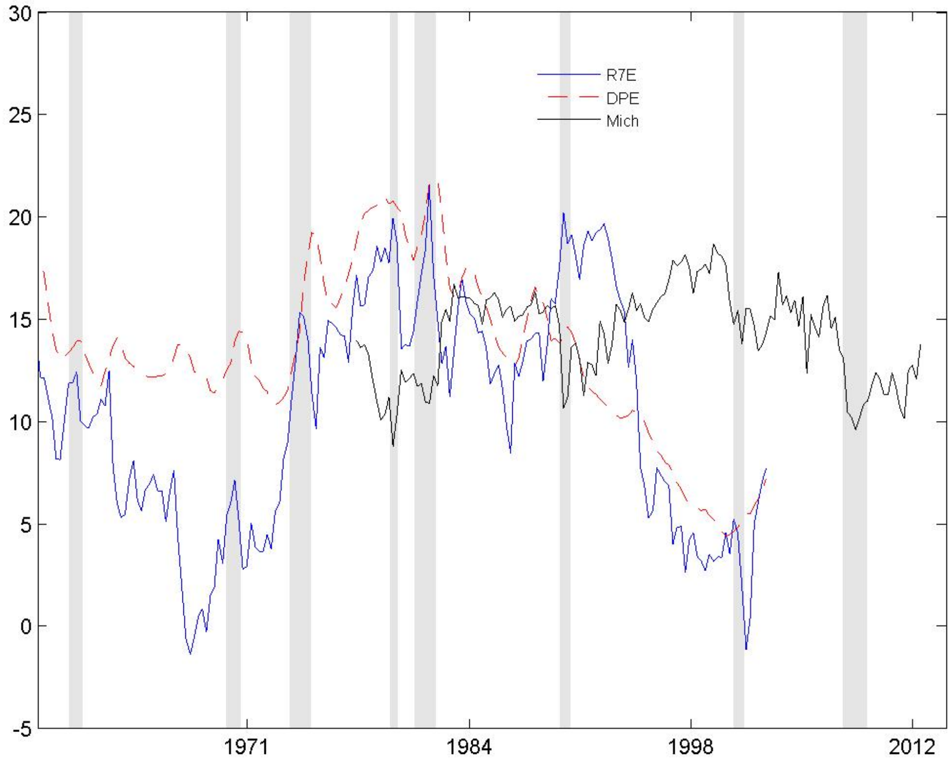
Note: FED is the effective Federal Funds Rate, TB3 is the rate on 3 months Treasury-Bills, G10 is the 10-year treasury bond yield. Real Interest Rates are calculated using the GDP deflator (GDPDEF).

Figure 8: Housing and Expectations



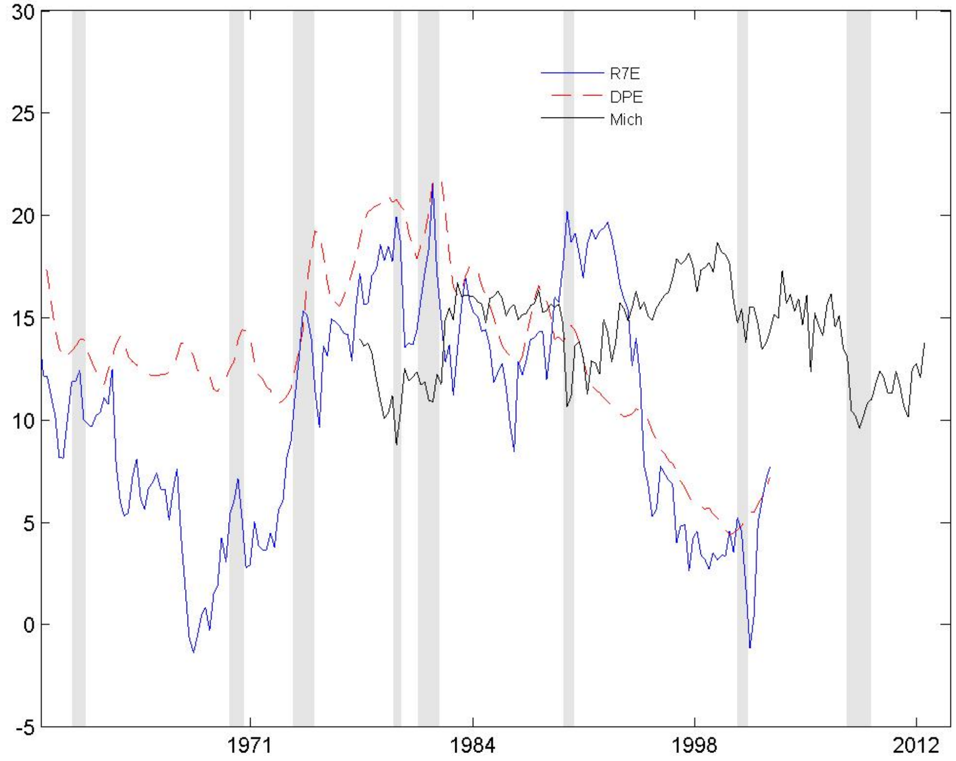
Note: R7H is the 7-year ahead real return on housing (the Case-Shiller index is used), DPH is the dividend price ratio on housing, and Mich is the Michigan Index of Consumer Expectations.

Figure 9: Stock Market and Expectations



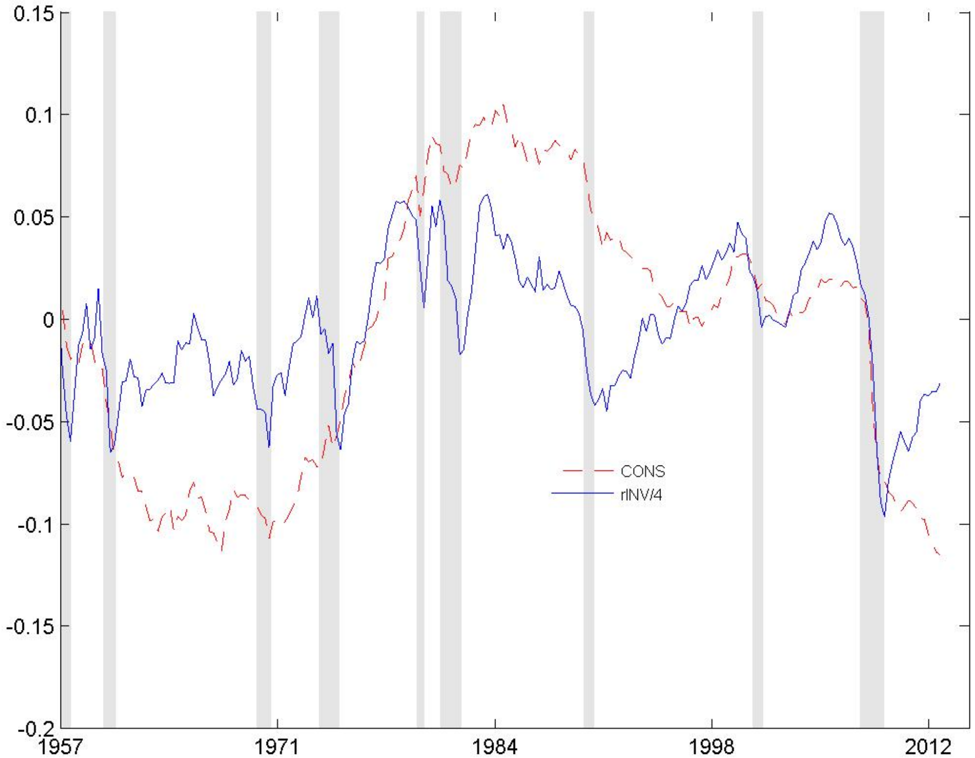
Note: DPE is the 7-year ahead real return on a stock index, and DPE is the dividend price ratio on stocks, and Mich is the Michigan Index of Consumer Expectations.

Figure 10: Real GDP and Real Wages



Note: RGDP is real gdp, RWAGES real wages. RGDP, RWAGES are log deviations from a trend calculated using a HP filter of very high parameter (10^7).

Figure 11: Consumption and Investment



Note: CONS is personal consumption expenditures, rINV is real gross private domestic investment. RCONS, rINV are log deviations from a trend calculated using a HP filter of very high parameter (10^7).

Chapter 4

House Prices Drive Current Accounts: Evidence from Property Tax Variations

This chapter was written with Thomas Grjebine, PhD Student at Sciences Po.

Abstract

We study the causal link between house prices and current accounts. Across time and countries, we find a very large and significant impact of house prices on current accounts. In order to rule out endogeneity concerns, we instrument house prices for a panel of countries, using property tax variations. A 10% instrumented appreciation in house prices leads to a deterioration in the current account of 1.7% of GDP. These results are very robust to the inclusion of the determinants of current accounts. Following a house price increase, private savings decrease, through wealth effects rather than consumer-finance based mechanisms, while non-residential investment rises through a relaxation of financing constraints for firms.

Keywords: Current accounts

JEL classification: F32, F36, F40

Acknowledgements: We especially thank Emmanuel Farhi and Philippe Martin for their advice and suggestions. We also thank Klaus Adam, Pol Antràs, Fernando Broner, Markus Brunnermeier, Benjamin Carton, Nicolas Coeurdacier, Arnaud Costinot, Gita Gopinath, Pierre-Olivier Gourinchas, Koen Joechmans, Guy Laroque, Albert Marcet, Adrien Matray, Thierry Mayer, Marc Melitz, Enrique Mendoza, Eric Mengus, Jonathan Parker, Jean-Marc Robin, Kenneth Rogoff, Esteban Rossi-Hansberg, José Scheinkman, Hyun Song Shin, David Sraer and seminar participants at Harvard University, MIT, Princeton University and CEPII for their comments and remarks. François Geerolf thanks Ecole Nationale des Ponts et Chaussées for funding.

Introduction

In a speech at the Annual Meeting of the American Economic Association, on January 3, 2010, Chairman Ben Bernanke presented a scatterplot showing a negative relationship between changes in current account and changes in real house prices between 2001 and 2006, in a cross-section of 20 advanced economies: "This simple relationship requires more interpretation before any strong conclusions about causality can be drawn; in particular, we need to understand better why some countries drew stronger capital inflows than others." This paper takes up Bernanke's proposal to investigate the causal relationship between house prices and current accounts. A better understanding of the determinants of current accounts is key in many policy debates such as global imbalances, or the eurozone crisis.

We show that house prices are a key determinant of current accounts, using a new instrumental variable for house prices (a property tax), that varies across countries and time. Our identification strategy relies on the fact that property tax changes are driven by local politics rather than macroeconomics, so that they are orthogonal to macroeconomic factors which might otherwise determine the current account. To the best of our knowledge, we are the first to instrument house prices in a panel of countries. This is important, since no previous paper has been able to rule out that expected positive productivity shocks would drive both house price growth and current account deficits; or that the causality would go the other way around, from capital inflows to house price booms. In contrast, we treat very carefully the business cycle dimension of house price movements.

The IV estimation yields similar estimates as the OLS estimation: a 10% increase in house prices yields to a deterioration in the current account of 1.7% of GDP. This is an economically very large effect, as the standard deviation of house prices is 30.4% in the whole sample, while that of current accounts is 4.89% of GDP. The variance decomposition therefore displays a very high explanatory power of house prices on current accounts. Moreover, in contrast to the previous literature using OLS or VAR techniques, our sample contains the universe of available country-year data for house prices and current accounts; our conclusions are therefore valid across 34 countries and between 1970 and 2010.[1]

We investigate empirically which theoretical mechanisms are at the source of the causal relation between house prices and current account deficits. We decompose the current account into four components which we analyse separately: private and public savings, residential and non-residential investment. Most notably, private savings decrease, but this is not the consequence of the availability of home-equity extraction, nor of high Loan-To-Value (LTV) ratios. Non-residential investment increases more in countries where the private sector is more credit constrained, thus suggesting that firms use real-estate assets to obtain financing, as corporate finance with asymmetries of information suggests. This is consistent with firm-level evidence from Chaney et al. (2012).

Related literature. We shall not review here the very vast literature on the current account, which comprise both the intertemporal approach (surveyed in Obstfeld and Rogoff (1995)) and the complete markets business cycles approach. The theoretical mechanism behind our empirical analysis is closer to Caballero et al. (2008*b*), as it emphasizes the role of asset supply in shaping current account patterns. Closer to our paper, many commentators

[1] Other papers, more theoretical in scope, also present evidence only for the last episode of the 2000s. See, Ferrero (2012), or Adam et al. (2011). In contrast, we use all available data on house prices and current accounts. For example, our OLS regression uses 833 country-year observations, and our IV regression uses 769, while existing work has relied more on less than 30 observations.

outside academia indeed have noted that the countries which experienced the worst housing booms were also those which ran current account deficits during the run-up in house prices. This observation is difficult to interpret because both house prices and current accounts can be expected to be affected by the business cycle, as the international RBC literature would suggest in particular. Some academic papers have started to address this issue more rigorously, but most explorations of the relationship are theoretical, and motivated the particular circumstances of the years 2000-2007. In Ferrero (2012), a shock to borrowing constraints is shown to be able to generate both house price increases and current account deficits. In the same theoretical vein, Adam et al. (2011) show that different expectations about asset prices can generate housing booms and current account deficits in those countries which are bullish about housing. Those are only two examples in a longer series of theoretical papers, which all use rough cross-correlation of cumulative increases in house prices and deterioration of net foreign asset positions as illustrative examples. This is also the case of Laibson and Mollerstrom (2010), in which (behavioral) asset price bubbles help explain the cross-country correlation between 2000 and 2006. There is a limited number of paper which look at the issue empirically. Aizenman and Jinjarak (2009) use data from $1990-2005$, and favor the reverse causality explanation. Their identification strategy relies mostly on Granger causality, and an instrumental strategy using real exchange rates or old dependency ratio to instrument current accounts.[2] The direction of causality has also started to be discussed separately for the US in the recent period : Favilukis et al. (2012) argue that changes in international capital flows played, at most, a small role in driving house price movements in the last fifteen years in the US, which is consistent with the conclusion of our paper. Some papers have also used structural VAR model for specific countries, or for a subsample of OECD economies, among which Fratzscher (2010) and Punzi (2007)). For example, Fratzscher et al. (2010) analyze the role of asset prices in comparison to other factors, in particular exchange rates, as a driver of the US trade balance. Gete (2010) shows that housing demand shocks identified in a SVAR model help to explain the trade balance in a sample of OECD economies.

Outline. The rest of the paper proceeds as follows. In Section 1, we present the database we have constructed on house prices and current accounts which, to the best of our knowledge, we are the first to use in a comprehensive way. We use HP-filtering to avoid spurious regression problems and compute HAC (heteroscedastic and autocorrelation) robust coefficients, since house prices and current accounts display some serial correlation. In Section 2, we present our OLS results, controlling for determinants which have been previously used in the literature, and using country fixed-effects. In Section 3, we present our Instrumental Variable results, which are not significantly different from OLS results. We use property taxes as an instrumental variable for house prices. We discuss very carefully exclusion restriction, which is that those property taxes do not result from macroeconomic factors. Consistent with this hypothesis, the instrumental variable we use is not correlated to GDP (see column

[2]However, one might worry that real exchange rates are endogenous to current accounts, and old dependency ratios directly affect house prices. Moreover, they cannot reject reverse causality for the US and the UK, and even suggest a consumption channel in the United States: "The US findings may be a case of a large real estate market in a large country, "driving" the business cycles...To the extent that it does, this finding might suggest that increased perceived wealth drives up prices and also drives up consumption and current account deficit." (p85-86) In this paper, we find evidence for this channel in the average country.

(1) of Table 10). In Section 4, we decompose the current account between public savings, private savings, residential investment and non-residential investment to understand better the channels through which house prices impact the current account. In Section 5, we analyse different theories of house price and current-accounts comovements. In Section 6, we perform a simulation exercise to understand how far one can go towards explaining current accounts with changes in national house prices. Finally, in Section 7, we perform some robustness checks.

1 Data and estimation technique

Data. We construct a yearly house price database for 34 countries[3] for the period 1970-2010. We have 833 observations in total for the pair house prices / current accounts (average per country: 29 years). The data for house prices was drawn from a number of different sources[4]. We notably use the property price statistics from the Bank for International Settlements which cover a large number of countries but only for a short period of time. To complete the database, we then bring together data from various national sources (central banks, national statistical agencies, etc.). There are issues of comparability across time and countries of this house price data: house prices sometimes refer to the price of residential structures in several big cities only. However, house prices are very correlated in the same country as we show in Appendix 3. Data for the current account are taken from the OECD statistical database.

The main specification of our paper is:

$$CA_{it} = \alpha H_{it} + \beta X_{it} + \delta_i + \nu_t + u_{it}.$$

CA_{it} and H_{it} are current accounts and house prices of country i in year t respectively. More precisely, CA_{it} denotes the current account as a percentage of GDP. H_{it} denotes an index of real house prices (that is, deflated by the CPI), in base 2005. X_{it} are controls for current accounts.[5] Following the literature on the current accounts, we will alternatively use Public sector surplus, Initial Net Foreign Asset Position, Relative income, the square of relative income, young relative dependency ratio, old relative dependency ratio, financial deepening, an oil dummy, real interest rates, real exchange rates. Note that some of these controls are clearly endogenous variables, jointly determined with current accounts. For example, real exchange rates, relative income, or interest rates are clearly jointly determined with current accounts. However, we will use them in some regressions, in order to compare our results to the existing literature. δ_i and ν_t are country and year fixed-effects. Country-fixed effects are included in all the regressions of this paper, and enable us to identify the effect of house prices on current accounts from the time-series dimension. We therefore control for any unobserved factor that may lead countries to have both high house prices and current account deficits.

[3] Our sample comprises Australia, Austria, Belgium, Canada, China, Czech republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, The Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, South Africa, Spain, Sweden, Switzerland, United Kingdom, the United States.

[4] A precise description of the database is provided in Appendix 1.

[5] A precise description of all the variables is provided in appendix 1.

Country-fixed effect also control for the fact that house price indices may not be comparable across countries, so that we are only left with interpreting the difference from the country-mean. Finally, we also add year fixed-effects in robustness check tables (Table 17 to Table 22).

Stationarity problems. Due to data limitation on housing prices, most of the economies we consider are advanced economies. A first problem with regressing current accounts on housing prices is that current accounts have a downward trend (advanced economies tend to borrow from emerging countries on aggregate), while house prices have an upward trend. We therefore use a HP-filter with a smoothing parameter of 400 to detrend our data, to remove the very low-frequencies.[6] Using augmented Dickey-Fuller and Phillips-Perron tests, we can then reject the hypothesis that current account series contain a unit root. Moreover, after regressing current accounts on house prices, we can reject the null hypothesis that residuals contain a unit root at reasonable confidence intervals, for all series in which we have a sufficiently large sample. Therefore, we are confident that we do not have spurious regressions problems.

Estimation technique. Since both current accounts and house prices are serially correlated, we must be careful to use robust estimation procedures, or we would be overestimating the precision of our coefficients. In this paper, we only present standard errors which are robust to heteroscedasticity and autocorrelation (HAC). We use the Bartlett kernel-based (or nonparametric) estimator, also known as the Newey and West (1987) estimator. We use a bandwith of 2, which leads that to the inclusion of autocovariances up to 1 lag. Note that automatic lag selection as in West (1994) is not available here since we use panel data. However our result are robust to different choices, for example inclusion of 2 lags. See Hayashi (2000) for more on GMM estimation with serial correlation.

2 OLS Results

The baseline regression yields the estimates displayed in Table 1. The correlation is very significant at the 1%. According to the simplest specification (column (1) of Table 1), an increase in house prices of 10% is associated with a deterioration of the current account of about 1.06% of GDP. The explanatory power of this regression is high: $R^2 = 18.1\%$ with house prices alone. Moreover, adding our house price variables to usual determinants of current accounts increases the R^2 by more than 13% (compare column (3) to column (2) in Table 1).

In columns (2), (3), (4) and (5) of Table 1, we follow the literature on the current account to compare the explanatory power of house prices with other variables usually put forward in the literature (see Chinn (2003) and Chinn and Ito (2007), and Obstfeld (2012) for recent references). In columns (2) and (3), we add the following variables:

(i) *Public surplus.* This corresponds to yearly public primary surplus, as a percentage of GDP. The intuition is that public borrowing increases overall borrowing from abroad, which

[6] Our results carry on when using first differences instead of a HP filter. We discuss the choice of the HP filter parameter in robustness checks in Section 7. The relationship between smoothing parameter and frequency under which data is kept is $\lambda = \frac{1}{\left[2\sin\left(\frac{\pi}{f}\right)\right]^4}$.

can increase current account deficits. Note however that in a ricardian world, this must be offset by more private savings.

(ii) *Relative income* (and the square of relative income). This is a way to control for different stages of development. According to neoclassical theory, capital should flow from rich to poor countries where returns are higher.

(iii) *Relative dependency ratio.* The young/old dependency ratio determines how much the population must save for retirement. Note however that this depends on whether the pension system is funded or pay-as-you-go.

(iv) *Financial deepening.* It is more easy to finance current account deficits when the financial system is deep.

(v) *Initial net foreign asset positions.* From a buffer stock perspective, higher levels of initial net foreign assets should be associated with subsequent lower current account balances.

(vi) *Oil dummies.* Oil exporters often build up reserves, which determines a positive current account balance - for example, Norway. Oil dummies were therefore added in current account regressions by researchers trying to assess the potency of the intertemporal approach to the current account.

Table 1: House prices and Current Accounts. OLS Regressions.

	(1) CA	(2) CA	(3) CA	(4) CA	(5) CA
House Prices	-10.61***		-9.887***		-8.310***
	(1.068)		(1.461)		(1.272)
Public Surplus		-0.103*	-0.0257	-0.0526	0.00542
		(0.0625)	(0.0580)	(0.0738)	(0.0706)
Initial NFA		9.182	8.705	5.924	7.053
		(9.205)	(7.485)	(9.096)	(7.783)
Relative income		-8.213	3.303	-4.603	3.459
		(8.186)	(7.959)	(8.855)	(9.011)
Relative income sq.		-74.24	-7.868	-230.5	-111.0
		(201.4)	(177.7)	(262.6)	(238.4)
Relative dependency ratio (Young)		-0.253	-0.451*	-0.428	-0.715***
		(0.255)	(0.241)	(0.261)	(0.249)
Relative dependency ratio (Old)		0.368	-0.0346	0.988**	0.370
		(0.445)	(0.440)	(0.473)	(0.474)
Financial deepening		0.00715	0.00972*	0.00681	0.00842
		(0.00661)	(0.00585)	(0.00606)	(0.00572)
Oil Dummy		-0.174	-0.345	0.341	0.0412
		(0.816)	(0.774)	(0.796)	(0.773)
Real interest rates				0.139	0.180
				(0.162)	(0.156)
Real exchange rates				-0.0580***	-0.0310**
				(0.0180)	(0.0152)
Observations	833	465	465	396	396
R^2	0.181	0.044	0.177	0.086	0.174

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects included in the regressions. Some of the controls are endogenous, notably real interest rates, real exchange rates and public surplus; but we include the controls which are common in the literature on current accounts.

However, note that many of these variables are somewhat endogenous - for example, relative income may depend a lot on whether a country is opened to trade, hence on his current account balance. A take from Table 1 is that these 8 variables explain only 4.4% of the variance in the current account, which is quite low when compared with the 17.7% explained if we add house prices.

Pitfalls with the baseline regression. Both current accounts and house prices are equilibrium quantities, which are jointly determined. Therefore, there are several issues with the OLS regression which prevent an interpretion of this correlation in a causal sense, from house prices to current accounts. The first issue is reverse causality: it is sometimes argued that a current account deficit could facilitate financing, hence a housing boom in a country (see references in Introduction).

Second, there is potentially an omitted variable problem, since many factors could drive both house price booms and current account deficits. For example, the expectation of a productivity shock in the country could both lead the country to borrow from abroad to finance present consumption and investment, and lead to house price appreciation, if housing supply is not perfectly elastic.[7] This omitted variable would lead to an overestimation of α in absolute value. Another potential explanation would involve financial deregulation. This could lead at the same time to increased foreign borrowing, hence a current account deficit; while at the same time easing credit constraints on local borrowers, hence driving house prices up. This would also lead to an overestimation of α (in absolute value).

Third, there is a clear problem of measurement errors in house prices. This is another reason to use an Instrumental Variable approach.

3 IV

A key contribution of this paper is to propose a new instrument for house price changes. Our instrument is property taxes as a percentage of total taxes (we will also be using other scaling variables). Because of capitalization, unexpected increases in property taxes are immediately translated into a decrease of house prices. Of course, an ideal variable would be constituted by a single flat tax rate, which would be levied on all estates, differ across countries, and change over time. However, taxes are highly multidimensional, nonlinear, with several brackets, and exemptions below a certain threshold. We therefore use the share of revenues brought about by property taxation in total taxation of a country. These data are produced by the OECD. A very important element of our taxation series is that property taxation essentially uses fiscal values (as opposed to market values) which are rarely revised to reflect market values[8]. Since we will observe a negative coefficient in the first stage, this will not be an issue: if anything, the more frequent revision of fiscal values towards market values would only weaken our first stage instrumentation, and go against our results.

[7] Ownership of housing is usually tied to the ownership of land, which in most countries is available in inelastic supply.

[8] We describe in Table 26 the frequency of revision of cadastral values for our sample of countries.

3.1 Data

The taxation variable we use comes from OECD Revenue Statistics. We use a particular sub-heading: recurrent taxes on immovable property. This sub-heading covers taxes levied regularly in respect of the use or ownership of immovable property. Since all the details of this tax are important, let us quote the *Revenue Statistics* in full length: "these taxes are levied on land and building, in the form of a percentage of an assessed property value based on a national rental income, sales price, or capitalised yield; or in terms of other characteristics of real property, such as size, location, and so on, from which are derived a presumed rent or capital value. Such taxes are included whether they are levied on proprietors, tenants, or both. Unlike taxes on net wealth, debts are not taken into account in their assessment." As already mentioned, an important feature of the tax we use is how its tax base is assessed, and in particular that it is not endogenously affected by house prices. Otherwise, it would be difficult to measure the negative impact of tax collection on house prices. By contrast, we estimate a negative relationship between our taxation variable and house prices.[9] The possible dependence of our taxation variable on market prices is therefore not sufficiently important to overturn this negative correlation, and this effect, if existent, would go against our conclusions.[10]

This property taxation variable is available at OECD as an absolute amount of collected taxes, as a percentage of GDP or as a percentage of total taxation revenues. We use property taxation as a percentage of total taxation receipts, because we want to capture variations in property taxation that keep total tax receipts constant, since changes in total tax receipts could impact the current account directly through government borrowing. A problem with using our tax variable as a percentage of total taxation is that real-estate property taxation variations could be driven by an increase in the value, hence in the share of other taxes following an increase in house prices. However, we check that this is not a concern. First, 95% of changes in our taxation measure come from an increase in the amount collected by property taxes, not from an increase in total taxes (in frequency terms). Moreover, in the Robustness section 7, and in particular in Table 14, we show that choosing other scaling variables for property taxes does not alter the results in any way. Instead of total taxes as a a scaling variable, we use taxes as a percentage of GDP, taxes as a percentage of Consumption or Investment, which all yield similar results.[11]

3.2 Regressions

The 1st stage equation. We use Two stage least squares (2SLS), with exogenous variation of real-estate property taxation T_{it} as an instrumental variable for house prices in the first

[9] This explains why we cannot use as instrumental variable non-recurrent taxes (real estate capital gain taxes, transaction taxes) as they are are endogenously affected by house prices (Table 9).

[10] A similar line of reasoning would argue that housing values as a basis of estate taxation are sometimes reassessed, and that this would also lead the taxation share as a function of GDP to be endogenous to house prices. However, once again, this would go against our conclusions: our instrument would be far more powerful and negatively related to house prices, if we divided it by house prices themselves.

[11] This method of using many different scaling variables is very common in the empirical finance literature, where dividends also need to be scaled, for example for estimating asset pricing equations - and where several scaling variables such as price or earnings are used to guarantee exclusion restriction.

stage. That is, the price of housing is given by the iteration equation:

$$H_{it} = \frac{H_{it+1}}{1+r} + R_{it}(T_{it}) - T_{it}.$$

The price of housing is the actualized resale price of housing tomorrow $\frac{H_{it+1}}{1+r}$ plus the rental dividend $R_{it}(T_{it})$ (either housing services provided to the owner occupying his home, or rents paid by the renter), diminished by the tax on property $T_{it}(H_{i0})$ with T_{it} an increasing function, whose tax base H_{i0} was set at the beginning of the period 0, once and for all (as this is the case for the countries we consider). In the remaining, we drop the dependence in H_{i0}. Note that the introduction of a tax T_{it} may change rents charged by owners, if housing supply is not completely elastic. In effect, the real-estate tax reduces the number of homes constructed in equilibrium, as agents want to avoid the burden of the tax, and this increases the equilibrium rents $R_{it}(T_{it})$. More precisely, partial equilibrium tax incidence analysis tells us that if $Q^d_{i\tau}(R_{i\tau})$ denotes the demand for housing at time τ as a function of its price (rental price $R_{i\tau}$), and if $Q^s_{i\tau}(R_{i\tau})$ denotes the supply of housing, then denoting the respective demand elasticity and supply elasticity by

$$\epsilon_D = \frac{R_{i\tau}Q^{d'}}{Q} \quad \epsilon_S = \frac{R_{i\tau}Q^{s'}}{Q}$$

then the net of tax rent is

$$R_{it}(T_{it}) - T_{it} = R_{it}(0) - \frac{\epsilon_S}{\epsilon_D + \epsilon_S} T_{it}$$

However, if housing supply is not completely inelastic that is $\epsilon_S \neq 0$, then the tax is not in the end borne by renters only, but also at least partly by proprietors. If we find in the data that our real estate tax has some negative effect on house prices, then this will mean that renters do not bear all the tax. Iterating forward (and ruling out rational bubbles) yields:

$$H_{it} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{1}{(1+r)^\tau} (R_{i\tau}(T_{i\tau}) - T_{i\tau}) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{1}{(1+r)^\tau} \left(R_{i\tau}(0) - \frac{\epsilon_S}{\epsilon_D + \epsilon_S} T_{it} \right).$$

For the last equality, we assume that the tax is set once and for all, and that changes are unexpected,

$$\forall \tau \in \{t+1, t+2, ...\}, \mathbb{E}_t T_{i\tau} = T_{it}.$$

We check in the first stage regression that this instrument is indeed related negatively to house prices, estimating the equation by least squares.

Magnitude of the 1st stage. This regression leads to the estimates displayed in Table 2. Note that the orders of magnitude of the change in house prices following an increase in property taxes are very high. A 1% increase in the share of property taxation in total taxes leads to a decrease in house prices of about 3.7%. Our instrumentation is very efficient, our first stage displays large and economically significant estimates. Our T-statistic for this 1st

Table 2: INSTRUMENTAL VARIABLE APPROACH

	(1)	(2)	(3)	(4)	(5)
1st Stage	House	House	House	House	House
Property tax	-3.697***	-3.611***	-3.587***	-3.394***	-3.216***
	(0.881)	(0.970)	(0.962)	(1.003)	(0.994)
Relative dependency ratio (young)		-0.0323***	-0.0304***	-0.0286***	-0.0305***
		(0.00730)	(0.00773)	(0.00716)	(0.00760)
Relative dependency ratio (old)		-0.0111	-0.00886	-0.0119	-0.0150
		(0.0158)	(0.0155)	(0.0157)	(0.0161)
Oil Dummy		0.00640	0.0172	0.00625	-0.0107
		(0.0252)	(0.0273)	(0.0278)	(0.0287)
Relative income			0.178***		
			(0.0640)		
Relative income sq.			3.125		
			(7.724)		
Real exchange rates				0.00361***	
				(0.000693)	
Financial Deepening					0.00180***
					(0.000405)
2nd Stage	CA	CA	CA	CA	CA
House Prices	-17.10***	-17.76***	-18.05***	-21.04***	-21.05***
	(4.588)	(5.084)	(5.063)	(5.661)	(5.886)
Relative dependency ratio (young)		-0.268	-0.255	-0.413*	-0.491**
		(0.234)	(0.227)	(0.239)	(0.238)
Relative dependency ratio (old)		-0.187	-0.167	-0.352	-0.0439
		(0.316)	(0.317)	(0.348)	(0.350)
Oil Dummy		0.393	0.722	0.406	-1.174
		(0.872)	(0.898)	(0.916)	(1.036)
Relative income			1.981		
			(2.412)		
Relative income sq.			-153.8		
			(200.4)		
Real exchange rates				0.0198	
				(0.0258)	
Financial Deepening					-0.0221
					(0.0165)
Observations	769	599	599	575	553
Cragg-Donald	23.50	19.00	19.08	17.90	14.54

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. House Prices are an indice of house prices, normalized at 1 in 2005. CA denotes Current Account. Country fixed effects included in the regressions. All series are HP-filtered.

stage is about 4.2 (higher than the Yogo rule-of-thumb), so that we do not suffer from weak instrumentation.

3.3 Exclusion restriction

For our instrument to introduce purely exogenous variations in house prices, property tax changes must not result from an omitted third factor, like economic conditions (GDP for example).[12] Our first argument in favor of exclusion restriction is that property taxes are usually set by local governments, and are not a tool used for macroeconomic policy. And indeed, unobservable factors such as GDP do not correlate at all with our instrumental variable (column (1) of Table 10). Moreover, we show that controlling for GDP (through our variable

[12] Falling GDP could lead for example to fiscal austerity, and higher property taxes.

of relative income[13]) does not alter our results in any significant way (see column (3) of Table 2).

Moreover, increases in total taxes, which correlate negatively with our instrument (column (4) of Table 10), could have effects on current accounts through increasing public surplus. However, this would go against our results, as it would both lead to current account surpluses, and be identified as increasing house prices in our sample.

A narrative approach: the example of Spain. A very important assumption for our IV strategy to be valid is that changes in the share of property taxation in the total taxes are uncorrelated with current accounts. We take the example of Spain where it is possible to shed light on four different property tax shocks over the last thirty years (Figure 1 and Figure 2)[14]. A first shock was the result of the decree law of 1979 which introduced an extensive package of measures for the reorganization of local treasuries, ranging from doubling the base of some property taxes (the Urban Land Tax) and the subsequent revision of all cadastral values. This decree law authorized gradual increases in property taxation, in particular with the law of 1983, whose consequence was a gradual decrease of house prices and an improvement of the current account. The reason for this change (reorganizing local treasuries) is likely to be orthogonal to other macroeconomic factors. A second shock was a sentence of the constitutional court of 1985 which overturned the law of 1983 and stopped the permanent increase in property taxation that had started in 1979. It resulted in an increase in house prices. Once again, it is very likely that the sentence of the constitutional court was orthogonal to other macroeconomic factors in the country. A third policy shock was the consequence of a law of 1987 which enabled local authorities to increase property tax rates. This possibility was first used in 1991 after the municipal elections. Between 1991 and 1993 local authorities showed a high level of activity, increasing rates annually from 0.588 in 1990 to 0.664 in 1993. This explains that the increase in property taxation was gradual in this period. These increasing rates were largely attributable to the absence of cadastral value revisions in this three-year period. When revisions were resumed effective 1 January 1994, we observe that the average rate went down that year to 0.658, and the house price decline stopped. Finally, the fourth policy shock was the consequence of a new tax reform at the end of 2006 which was aiming at preventing tax frauds. In practice, the new law led to an increase of the local property tax (Impuesto de Bienes Inmuebles).

Testing for weak instrument. We have already discussed (by means of an example) the fact that the effect of taxation on house prices is first order. We also check that the Angrist-Pischke multivariate F-test of excluded instruments is about 18, so that our instrument is not a weak instrument.[15]

IV results. Using the property tax as an instrument for house prices with (**??**) as a first stage gives the results in Table 2. Looking at the column (1) of the 2nd stage, we get

[13] In the robustness checks, we control also by other measures of GDP. Results are similar (Table 11).

[14] A precise description of the 4 shocks and of their consequences is provided in Appendix 4.

[15] Our IV strategy also passes underidentification tests (the Kleibergen-Paap rank LM statistic is 18.98 for the main specification of column (1) in Table 2), and weak identification tests Cragg-Donald Wald F statistic is 24.46, and Kleibergen-Paap Wald rk F statistic is 17.58. In the second-stage, the underidentification test Kleibergen-Paap rk LM statistic is 18.979 and the Cragg-Donald Wald identification test F statistic is 24.460, while the Kleibergen-Paap rk Wald F statistic is 17.581.

Figure 1: INSTRUMENT, HOUSE PRICES AND CURRENT ACCOUNTS IN SPAIN

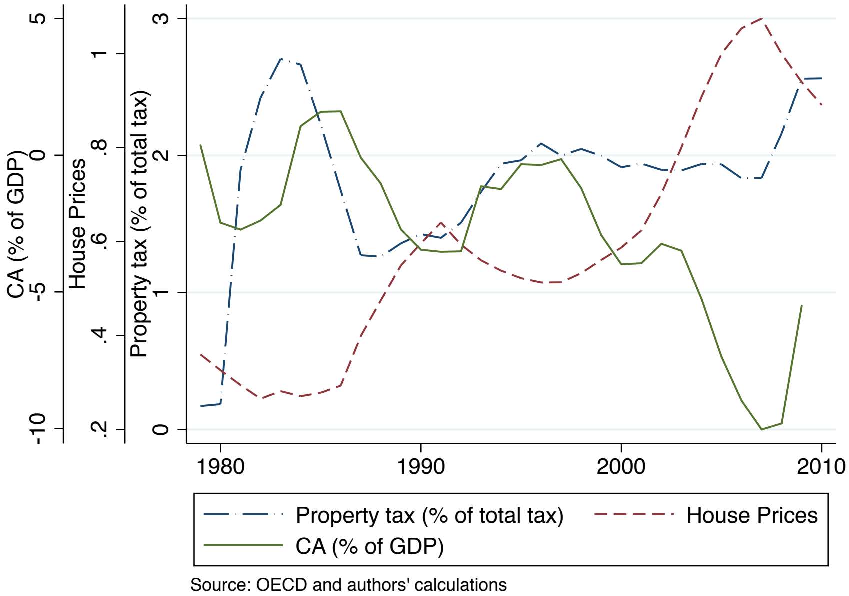
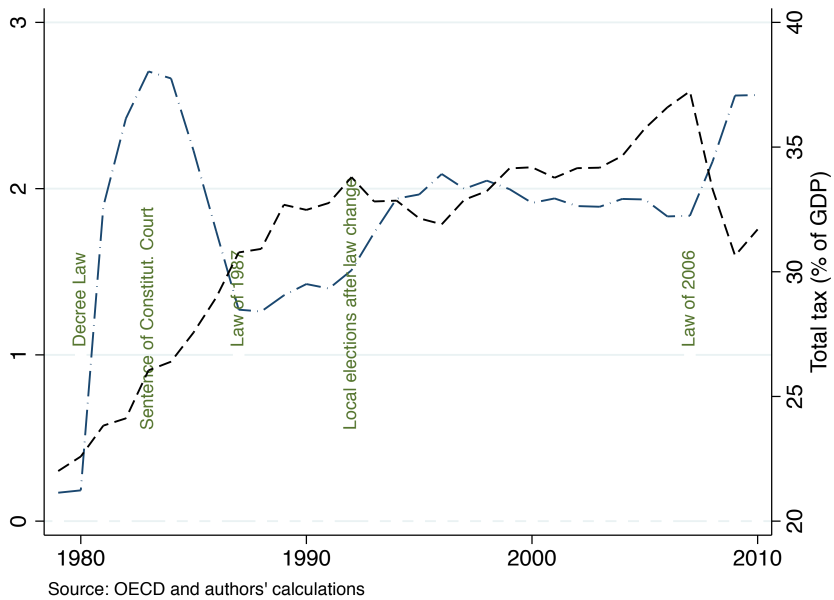


Figure 2: PROPERTY TAXES AS A % OF TOTAL TAX (BLUE), TOTAL TAX AS A % OF GDP (BLACK), AND POLICY SHOCKS in Spain



that a 10% increase in house prices yields to a deterioration in the current account of 1.7% of GDP . Note again that we present standard errors which are robust to heteroscedasticity and autocorrelation (HAC), use the Bartlett kernel-based (or nonparametric) estimator, also known as the Newey and West (1987) estimator, with a bandwith of 2. This estimation by GMM (which for simplicity, we simply call "Instrumental Variable", even though it is a GMM generalization of IV) is not significantly different from that obtained by ordinary least squares.

Comparing column (1) (2nd stage) in Table 2 with column (1) in Table 1, we interpret the increase in the coefficient with respect to OLS (in absolute value) by the fact that house prices are mismeasured and that OLS estimates are therefore biased towards 0. This suggests also that reverse causality is not at work in the data (current account deficits do not generate higher housing prices).

What do we instrument? Back-of-the envelope calculations suggest that a 1% increase in property taxes (as a percentage of total taxes) represents about 0.4% of GDP (assuming a tax take at 40% of GDP). However this change is known not to be permanent (perhaps for political economy reasons), because real estate taxes as a function of total taxes are not a random walk. Rather, tax cuts or rises approximately last about 5 years (estimating an AR(1) yields an autocorrelation coefficient $\rho \simeq 0.8$, or 2% of GDP). According to our first-stage regression estimates, and assuming rational expectations from the part of investors, a tax rise of 1% as a percentage of total taxes leads to a decrease in house prices of 3.7%, which is about 7.4% of GDP in capitalized losses (with a housing stock evaluated at 200% of GDP). There could be two explanations to this effect of taxes that goes beyond the fundamental effect. Either agents do not have rational expectations about the true data generating process governing taxes - for example taking tax changes as being permanent, even though they tend to mean-revert. Or our instrument may capture both fundamental and bubbly components of house prices[16].

4 Decomposition of the current account

Before testing different theoretical channels for explaining the causal relation we documented, we look more carefully at the components of the current account. In particular, we decompose the current account into four components: private savings S_p, public savings S_g (which together make up for total savings $S = S_p + S_g$), residential investment I_r and non-residential (business) investment I_b (which add up to total investment $I = I_r + I_b$). The current account equals $CA = S - I$. The results are displayed in Table 3.

House prices have a causal negative impact on private savings and a positive impact on non-residential investment.

Residential investment. The effect on residential investment is rather muted compared to that of non-residential investment, as an increase of 10% in house prices yield to increase of the residential investment rate of about 0.46% of GDP (Column (2) of Table 3C) . The IV estimate of this number is not significant, confirming that it is in any case a rather muted effect (Column (5) of Table 3C). The OLS result could be interpreted as a result of more expensive homes, which drives up construction volumes, keeping construction costs constant. For example, Spain in the 2000s witnessed a construction boom; new houses were built, often

[16]It is unclear what pushes people to become bullish at the sames time, but changes in taxes could be an element of this coordination. In particular, if there is competition between countries for being the locus for stores of value, taxes could be an element of this competition.

Table 3: DECOMPOSITION OF THE CURRENT ACCOUNT

	(1) OLS	(2) OLS	(3) OLS	(4) IV	(5) IV	(6) IV
	Table A: Current Account = Savings - Investment					
	CA	Saving	Invest. .	CA	Saving	Invest.
House Prices	-10.35***	2.618***	14.07***	-17.66***	17.78***	38.29***
	(1.136)	(0.953)	(1.154)	(5.075)	(6.195)	(7.543)
Observations	721	721	721	721	721	721
R^2	0.156	0.020	0.305			
	Table B: Savings = Private Savings + Public Savings					
	Saving	Pr. Sav.	Pu. Sav.	Saving	Pr. Sav.	Pu. Sav.
House Prices	2.203***	-5.321***	8.375***	27.16***	-17.01***	46.38***
	(0.855)	(0.925)	(1.332)	(9.791)	(6.383)	(14.38)
Observations	621	621	621	621	621	621
R^2	0.016	0.073	0.113			
	Table C: Investment = Residential + Non-residential Investment					
	Invest.	R Invest.	NR Invest.	Invest.	R Invest.	NR Invest.
House Prices	11.49***	4.605***	6.829***	33.68***	1.475	32.17***
	(1.046)	(0.378)	(0.923)	(9.154)	(2.825)	(9.417)
Observations	591	591	591	591	591	591
R^2	0.273	0.365	0.134			

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects included in the regressions. All series are detrended using a HP-filter.

with imported capital, and that contributed to a deterioration of the current account. The take from our regressions is that this effect might be part of the story, but explains only a very thin part of it.

Non-residential investment. Less mechanic and more interesting is the rise in non-residential investment following house price increases. According to Column (3) of Table 3C, non-residential investment increases by 0.68% of GDP following a house price increase of 10%. Using the Instrumental Variable estimator yields a much higher estimate of 3.21% of GDP (Column (6) of Table 3C).

Private savings. Private savings decrease when house prices increase according to the instrumental variable specification: about 1.7% of GDP for each 10 points rise in the house price index (Column (5) of Table 3B). This is the well-known consequences of housing booms, and the much commented "wealth effect".[17] In light of the effect of house prices on public savings, it could also be that households are partially ricardian.

Public savings. This issue would require further explorations. Mostly, public savings are not the result of market forces, or automatic stabilizers put in place in good times to prepare for bad (public insurance).

[17] Note however that this "wealth effect" is far from obvious theoretically, as housing is both an asset and a necessary outlay. In this respect, housing wealth is very different from stock-market wealth. Anticipating a bit, the rise of consumption following increases in housing wealth could be interpreted as an evidence for a rational bubble.

5 The role of credit constraints

Our data enables us to test two different channels through which house prices affect current accounts.

Table 4: Consumer and firm credit constraints

Table A	(1) CA (OLS)	(2) CA (OLS)	(3) CA (OLS)	(4) CA (OLS)	(5) CA (OLS)	(6) CA (IV)	(7) CA (IV)
House Prices	-13.93***	-11.55***	-18.21**	-12.07***	-11.18***	-17.20**	-101.9
	(1.999)	(1.528)	(7.571)	(1.560)	(1.684)	(8.702)	(314.3)
House*LTV			0.0622				
			(0.0880)				
Observations	416	417	604	500	333	416	353
R^2	0.261	0.340	0.275	0.365	0.281		
LTV	$< 80\%$	$> 80\%$				$< 80\%$	$> 80\%$
Extraction				No	Yes		

Table B	Invest. (OLS)	R Invest. (OLS)	NR Invest. (OLS)	Invest. (IV)	R Invest. (IV)	NR Invest. (IV)
House Prices	7.017***	5.103***	1.935			
	(2.080)	(1.313)	(1.906)			
House*1/PCGDP	581.6***	50.76	529.3***	3,513***	123.9	3,371***
	(190.4)	(86.90)	(174.9)	(879.9)	(308.1)	(928.0)
Observations	664	664	664	664	664	664
R^2	0.477	0.429	0.354			

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and year fixed-effects are included. For LTV ratios, the threshold we use (80%) is the median.

The first channel is the consumer-financing channel. Many papers in the literature have emphasized the potential role of borrowing constraints for driving both an increase in foreign borrowing and a run-up of house prices. According to these papers, in the 2000s, the US experienced a decrease in credit constraints. At the same time, houses saw their collateral value increase and the United States borrowed more to the rest of the world.[18] Interestingly, our data enables us to test whether the relaxation of borrowing constraints might have triggered current account deficits, together with an increased value of housing collateral (for its collateral services). We use measured maximum Loan-to-Value (LTV) ratios and show the relationship between current accounts and house prices is no higher in countries with high LTV ratios than those with low LTV ratios. According the estimated displayed in columns (1) to (3) of Table 4A, and (6)-(7) of Table 4A, whether a country has low LTV or high LTVs makes no difference to the correlation. In columns (4)-(5) of Table 4, we show that the availability of home-equity extraction does not increase the relationship between those two

[18]Note that there are theoretical issues to this explanation: it is unclear why constrained consumers, or investors, could not previously sell their house for the whole of their value, instead of buying a home and then use this home as collateral. In Ferrero (2012), as well as other papers of the like, in particular Iacoviello (2005), there is no such issue since homeowning is necessary for consuming housing services - there are no renters. Second, financial liberalization started in the 2000s , but the relationship between house prices and current accounts is not confined to the last boom, or to advanced economies, so that this explanation cannot be an explanation for the correlation before that.

variables either. The consumer-financing channel does not seem to be a feature of our data.[19]

A second channel is the firm-financing channel. We test whether rising housing values help relaxing financial constraints for firms[20]. In order to assess whether investment rises more with house prices where financial constraints are more stringent, we use as a proxy for the potential tightness of credit constraints, the ratio of private credit to GDP. This is a standard measure of financial development in the finance-and-growth literature, and provides substantial time-series and cross-sectional variation in our panel (Aghion et al. (2010)). We construct an interaction variable between house prices and the ratio of private credit to GDP. The simultaneous influence of two variables is significant for total investment and non-residential investment, as columns (1) and (3) of Table 4B show in OLS and columns (4) and (6) of Table 4B show using IV. These results confirm that the effect goes through a relaxation of financing constraints for firms. It is interesting to notice that the interaction variable is not significant in explaining residential investment. Since it is not construction firms who are the final investors in residential structures, it does not matter whether construction firms are financially constrained. Furthermore, this is consistent with the fact that houses are much less entrepreneur-specific investments, and that information asymmetries creating the need for collateral are quantitatively very low in housing investment. One can compare our estimate to other estimates found in particular through microeconomic studies of firm investment, as in Chaney et al. (2012): in their study, the representative US corporation invests \$0.06 out of each dollar of collateral. If 10% of house price increases corresponds to 20% of GDP of collateral because the housing stock is equal to 2 times GDP, then Chaney et al. (2012)'s estimate would predict a macroeconomic effect on investment of about 1.2% of GDP, which is the same order of magnitude as both our OLS and our IV estimators.

6 Simulating Current Accounts

Movements in house prices can be due to many factors - risk aversion, expectational shocks (bubbles), etc. Taking these movements as given, we can recover the current account patterns which would be generated by our very parcimonious linear model. An argument in favour of considering house prices as the source of exogenous shocks is that taking Ordinary Least Squares or Instrumental Variable estimates yields very comparable estimates. There does not seem to be much more to the relationship between house prices and current accounts than these shocks to house prices.

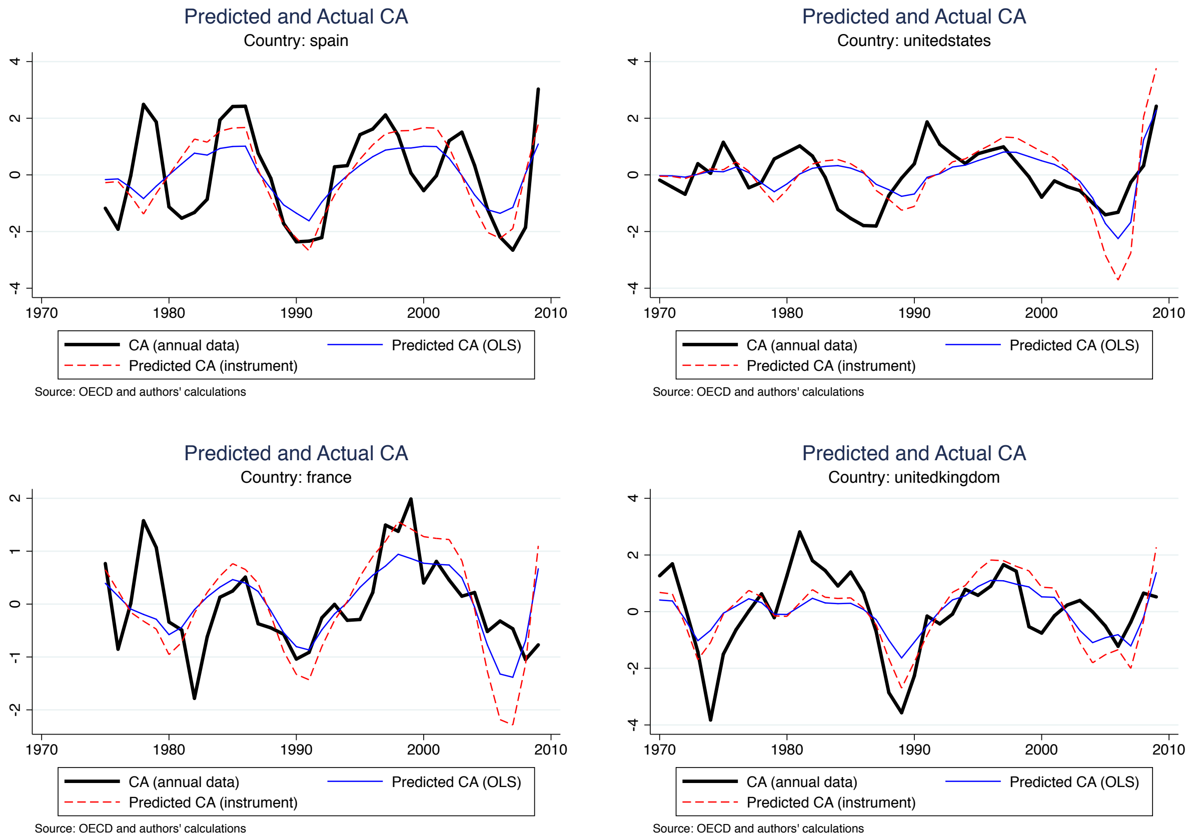
The results of this simulation exercise are summarized in Figure 3. For most countries, and in particular those which have been at the center of very important policy debates recently,

[19] The fact that home-equity extraction funds have been shown to be used for consumption in many microeconomic studies does not contradict our results in principle. Availability of home-equity extraction could just push more people into becoming homeowners even though they have high discount rates. These would have consumed nonetheless.

[20] Note however that this explanation does not explain jointly the rise in house prices and current account deficits, but only the fact that rising housing prices lead to current account deficits. In order to explain jointly the rise of house prices and current account deficits as in Ferrero (2012), one would need to assume that there was a firm-financing liberalization shock, which authorized more firms to take on loans backed by housing collateral.

such as Spain, France, Germany, the UK and the US, predicted patterns of the current accounts match actual ones reasonably well.

Figure 3: Simulated current accounts and actual ones



7 Robustness checks

For the sake of brievety, tables corresponding to robustness checks are at the end of the paper, before the Appendixes.

Granger causality. In this paper, we have used an instrumental variable approach to alleviate the issues of endogeneity and omitted variables. We also check in this section that Granger causality tests confirm that house prices cause current accounts and not the other way around. Table 5 shows that fitting simple VAR with either 1 lag or 2 lags[21] confirm this result: a positive shock to house prices does cause a deficit in the current account in the period after (columns (1), (3) and (5)) while capital inflows (a negative shock to CA) does not cause increases in house prices as can be seen in columns (2), (4), (6). One may note a very small effect of the second lag of capital inflows (only significant at 10%), but which goes in the other direction. Once again, the view that capital inflows cause housing bubbles seem refuted by the data. We have not pursued this empirical strategy in the remainder of the paper, even though it seems to yield the same conclusions qualitatively, because Granger

[21] To determine the number of lags, we use the Akaike Information Criterion (AIC) and the Schwarz' Bayesian Information Criterion (SIC/BIC/SBIC). For most countries, they indicate 2 lags.

causality is not *strictly* causality, and more importantly because the coefficients are impossible to interpret quantitatively. Since a very important take from our paper is that the house price variable we introduce is a very good predictor of Current Accounts, VAR techniques clearly would not lead us as far as the instrumental variable. We check also that our instrumental variable causes house prices. Table 6 confirms that property taxes cause house prices and not the other way around.

Public savings. In Table 7, we analyse more precisely why house prices are strongly positively correlated to public savings, and even cause an increase in public savings (see column(6) of Table 3B). Since public savings are less the results of market forces, investigation into the issue is more tentative. Our data seems to point to an effect of house prices through investment then unemployment[22]. We have established in section 5 that non-residential investment increased more consecutive to house price increases when countries were more financially constrained: this is reminded in column (2) of Table 7A. Using then our property tax as an instrument for investment in column (3) of Table 7A points to a decrease in unemployment following investment booms. In column (3) of Table 7B, we show that less unemployment is also associated with less spending by the government, which is intuitive, as a big part of welfare state entitlements come from unemployment benefits. This is reflected into overall government savings in column (2) of Table 7B. To sum up, our data explains the pro-cyclicality of public savings with respect to housing booms by an increase in investment leading to a decrease in unemployment. In contrast, when housing prices go down, investment also plunges because financial frictions increase and unemployment increases.

Falsification tests. In Table 8 and 9, we perform falsification tests using other taxes available from the OECD to instrument house prices. Since those taxes are not (in principle) related to housing, we should not be getting anything out of these exercises, which is what we verify in Table 8. In Table 9, we show that most other tax takes related to housing are positively correlated to housing prices. As housing prices go up, these tax takes mechanically increase. It is therefore not possible to isolate the negative impact that tax rates shocks have on housing prices. Once again, fiscal values used for property taxation are seldom revised, which enables us to estimate the negative effect tax rates shocks have on housing prices.

Examining exclusion restriction: more specifications. In Table 10, we show that the GDP is not correlated with the property tax. In fact measures of property taxation as a percentage of the total tax (column (1)) or as a percentage of GDP (column (2)) do not correlate with the GDP. So changes in property taxes do not have to do with the economic outlook. We show also that an increase in our instrumental variable does not imply increasing government revenues. Indeed, our property taxation variable correlates negatively with total tax revenues (column (5)).

Controlling by different measures of GDP. Our results do not depend on the measure of GDP used. In most tables, we control with relative income as it is the variable commonly used in the literature (notably in Chinn (2003) and Chinn and Ito (2007)). But our results are robust to other measures of GDP. In Table 11 , we show that we could have controlled by real GDP or real GDP per capita without changing the results of our instrumental strategy.

[22] We investigate more fully this mechanism in Geerolf and Grjebine (2013).

Other asset prices. One could wonder whether the negative relationship we uncover would not be true for other types of assets. In Table 12, we show that this correlation is not valid for equity prices. We use two variables to measure share prices. The first measure is Market capitalization (also known as market value). It is the share price times the number of shares outstanding as a percentage of GDP (source: WDI). The correlation between market capitalization and house prices is not significant (column (1) of Table 12). There is a slight difference when we use instead the other measure, Share prices (source: OECD). We find a very small negative relationship between share prices and current account variations, only significant at 10% (column (2) of Table 12). This very slight significance can itself be explained by the very strong correlation between house prices and share prices (column (3) of Table 12). If we first take the residual of the regression between houses price and share prices, and if we then run the regression between this residual and the current account, the relationship disappears. Intuitively, this fact can certainly be rationalized by the fact that contrary to most other assets, houses are geographically located assets. In contrast, differences in world share prices are arbitraged away in international capital markets. While share price cycles are strongly correlated at the world level, house prices are much less correlated: regressing share prices over year fixed effects in a panel of countries yields to a R^2 of 64%, while the same regression yields to a R^2 of only 31% for house prices.

Choice of HP filter parameter. The relationship we uncover in this paper is robust to several specifications of the cutoff frequency. Table 13 displays the result of our basic specification using different values for the HP-parameter. Any HP-filter parameter in the range $10 - 1600$ yields the same results with very good confidence intervals. There is some disagreement in the literature as to which filter to use for frequencies different from quarterly data - for quarterly data, a common practice in the literature is to use a parameter of 1600. We have used 400, as in Tomz and Wright (2007). Our results are robust to other lower proposed values of 6.25 (Ravn and Uhlig (1997)), 100 in Backus (1992) or higher, such as 1600. Note that this is not very important here, as we are interested only in first moments, not in second moments, for which the choice of the HP filter is more crucial - this is in fact, what the discussion in Ravn and Uhlig (1997) is all about - notably Backus (1992)'s claim that output volatility had increased after the Second World War. When choosing our smoothing parameter, we have only two requirements in mind: that it be not too small, because we are interested in medium term patterns of the data (not only those that occur at the quarterly frequency) - that is why we do not take up propositions in the lower range, and that it be not too high, because we want to remove the trend from the data (the lower frequencies) - long run growth, which we do not seek to explain - and because we do not want our series to be non-stationary, which would cause problems of spurious regressions.

Other scaling variables: using tax as a percentage of GDP. In Table 14, we show that using as an instrumental variable the share of property taxation as a percentage of GDP instead of using the share of this tax as a percentage of total taxes does not change the results. The results are also robust if we measure the property tax with other scaling variables, such as investment (column (2)) or private consumption (column (3)).

Country groupings. We also test whether the relationship we uncover in this article is specific to a certain type of countries, or whether it is robust across groups of very different countries. As Table 15 shows, the relationship is robust. The relationship is true in Euro or non-Euro countries (columns (2) and (3) of Table 15), and in low-income and high-income countries (columns (4) and (5)). This is also important as previous determinants of the current accounts were often specific to advanced or developing countries. Moreover, it is important to check for robustness that excluding several countries does not change the results in a significant way.

Credit constraints: further regressions. In Table 16, we run more regressions to examine the robustness of our findings in section 5. In columns (7) and (8) of Table 16A, we show that private savings are not more correlated to house prices in countries with high LTV ratios than in countries with low LTV ratios, further undermining consumer-financed based explanations of the correlation. In Table 16B, we check if our results on firm-credit constraints and collateral are robust to the inclusion or exclusion of country- and year- fixed effects.

Year fixed-effects. We did not include year-fixed effects in the baseline regression because we do not have the full sample of countries in our dataset.[23] But results and comments of previous sections are robust to the inclusion of year fixed-effects (Table 17 to Table 22). For example, a 10% (instrumented) appreciation in house prices leads to a deterioration of the current account of -2.4% (table 17, column (8)). First stage regressions of the instrumental strategy are also very robust to the inclusion of year fixed effects. A 1% increase in property taxes is associated with a depreciation of house prices of -1.9% (table 19, column (2)) . In the second stage of the IV strategy, the regressions are still very robust even with the inclusion of the current account controls and with year fixed effects (table 20, columns (4),(6), (8) and (10)). In tables 21 and 22, we check that our instrumental variables strategy is robust for explaining investment and saving with the controls and fixed effects. For instance, the second stage instrumental regressions are very robust for explaining investment even with the inclusion of the current account controls and with year fixed effects (table 21, columns (4), (6), (8) and (10)). In particular, 10% (instrumented) appreciation in house prices leads to an increase of investment of 4.7% (columns (4) and (6)).

Frequency of revision of cadastral values. In Table 23, we show that our results do not depend on the frequency of revision of cadastral values. In particular, the negative relationship between house prices and the property tax (first stage of the instrumental strategy) is no weaker in countries where fiscal values are reassessed at least every five years.

Decades. In Table 24, we show that our results are valid all over the last 40 years, and in each decade. House prices have a causal effect on current accounts not only in the last housing cycle (column (6)), but also in the nineties (column (5)), and before 1990 (column (4)).

Real Exchange Rates. In Table 25, we show that capital inflows driven by house prices could lead to exchange rate appreciation. This explains that house price increases are positively correlated with exchange rate appreciations (column (2)), and that in the IV,

[23] These fixed effects would capture the current account that our sample countries collectively run with the rest of the world. When house prices in our sample are above trend on average, we can capture that our sample countries are running deficits with the rest of the world.

real exchange rates are not significant in explaining current accounts (column (1)). Granger causality tests confirm that house prices cause real exchange rate fluctuations (column (3)).

Conclusion

In this paper, we establish that house prices are an important factor in the determination of current accounts, probably the variable with the largest explanatory power of current accounts over the last 40 years. Our new instrumental variable for house prices allows us to control for potential reverse causality or omitted variable problems. An instrumented increase in 10% of house prices leads to a deterioration in the current account of 1.7% of GDP. Not only are house prices strongly significantly correlated to current accounts, and the coefficient precisely estimated; but this point estimate is economically very large, suggesting that house prices are the main factor determining current accounts.

We investigate empirically which theoretical mechanisms are at the source of this causal relationship. We decompose the current account into its components that we can analyse separately. Private savings decrease following house price increases, but consumer-financing explanations are not consistent with the data, as this effect is not greater in countries where financing is easier. In contrast, we show a large increase of non-residential investment following house price increases, which we demonstrate goes through a collateral effect. Housing collateral therefore plays a big role in driving the correlation between house prices and investment, confirming the predictions of Caballero and Krishnamurthy (2006) in particular.

We then simulate current accounts. Taking house price shocks as given enables to recover extraordinarily well movements in current accounts. There are many reasons to think that house prices could have a life of their own: changes in risk aversion, in the stochastic discount factor, etc. Among other stories, our results are consistent with a view of (country-specific) expectational shocks on housing as a driving force for changes in asset supply. Real-estate bubbles are both theoretically plausible, as short-sales constraints are very high on real-estate, so that pessimists are at corner and cannot express a negative opinion (as in Harrison and Kreps (1979)); and a potential participant in banking crises (see Reinhart and Rogoff (2008)). The relationship between increases in asset supply and current account deficits would then be similar to Caballero et al. (2008*b*), in which a decrease in asset supply a China (corresponding to a relative increase in asset supply in the United States) leads to current account deficits in the United States.[24] Similarly, country-specific house price bubbles could increase asset supply which leads to deficit. Bubbles would move from one real-estate market to the next, as in Caballero et al. (2008*a*), leading to capital flows. Those bubbles would decrease private savings, as in Tirole (1985); and increase investment through alleviating financial constraints as in Farhi and Tirole (2011).

The policy implications of our results are potentially important. Current account imbalances were on top of the macroeconomic research agenda in the year 2000s, when the US were running unprecedented current account deficits (up to 6% of GDP). If once admits that house prices had overshooted their long-run level by about 20% (this is a rather conservative

[24] It is very important for this result that the environment be non Ricardian, or an increase in asset supply would lead to an offsetting increase in asset demand.

estimate), then our results would suggest that house prices contributed to these deficits up to 3.4% of GDP. But after the financial crisis, understanding the determinants of the current account is no less central (see Obstfeld (2012)). In particular, since current account capital flows have shown to be a major destabilizing factor in the fate of the euro, we believe our paper can bring important insights in the context of the Eurozone crisis.

Finally, the welfare implications of potential house price bubbles are not clear. While rational bubbles solve the problem of dynamic inefficiency (as in Tirole (1985)), housing bubbles can come at cost, triggering capital flow reversals as in Caballero and Krishnamurthy (2006). This is an interesting route for future empirical and theoretical research.

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Tables: Robustness checks

Table 5: Granger causality

	(1) CA	(2) House Prices	(3) CA	(4) House Prices	(5) CA	(6) House Prices
Current Account (L1)	0.435***	0.000322	0.466***	-0.000595	0.365***	0.00171
	(0.0976)	(0.00132)	(0.0942)	(0.00125)	(0.0586)	(0.00122)
Current Account (L2)	-0.160	0.00154	-0.174*	0.00208*		
	(0.103)	(0.00134)	(0.102)	(0.00123)		
House Prices (L1)	-10.82***	1.278***	-9.981***	1.189***	-6.040***	0.780***
	(2.809)	(0.0515)	(2.883)	(0.0496)	(1.344)	(0.0385)
House Prices (L2)	5.733**	-0.630***	4.961*	-0.547***		
	(2.816)	(0.0530)	(2.849)	(0.0466)		
Observations	673	673	673	673	697	697
R^2	0.276	0.727	0.335	0.780	0.291	0.682
Country FE	No	No	Yes	Yes	Yes	Yes
Year FE	No	No	Yes	Yes	Yes	Yes

Notes: Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Series are HP filtered.

	(1) Saving	(2) House	(3) Saving	(4) House	(5) Investment	(6) House	(7) Investment	(8) House
Saving (L1)	0.774***	0.000386	0.786***	0.00190				
	(0.0455)	(0.00147)	(0.0461)	(0.00147)				
Saving (L2)	-0.382***	0.00259	-0.328***	0.00291*				
	(0.0440)	(0.00162)	(0.0460)	(0.00158)				
House Prices (L1)	5.127***	1.266***	2.745**	1.179***	13.95***	1.270***	9.455***	1.165***
	(1.063)	(0.0491)	(1.066)	(0.0453)	(1.606)	(0.0578)	(1.465)	(0.0533)
House Prices (L2)	-7.222***	-0.645***	-3.683***	-0.562***	-13.75***	-0.642***	-8.514***	-0.554***
	(1.220)	(0.0518)	(1.176)	(0.0430)	(1.520)	(0.0550)	(1.368)	(0.0471)
Investment (L1)					0.640***	0.000839	0.706***	0.00263*
					(0.0614)	(0.00158)	(0.0586)	(0.00159)
Investment (L2)					-0.263***	-0.000608	-0.263***	-0.00128
					(0.0611)	(0.00177)	(0.0628)	(0.00160)
Observations	673	673	673	673	673	673	673	673
R^2	0.468	0.728	0.575	0.783	0.516	0.725	0.631	0.780
Country FE	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	No	Yes	Yes	No	No	Yes	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Series are HP filtered.

Table 6: GRANGER CAUSALITY (CONT.)

	(1) Property Tax	(2) House Prices	(3) Property Tax	(4) House Prices
Property Tax (L1)	0.892***	-0.00758**	0.638***	-0.0107**
	(0.0649)	(0.00356)	(0.0693)	(0.00446)
Property Tax (L2)	-0.373***	0.00226		
	(0.0502)	(0.00300)		
House Prices (L1)	-0.364	1.195***	0.192	0.754***
	(0.302)	(0.0456)	(0.198)	(0.0341)
House Prices (L2)	0.466*	-0.570***		
	(0.255)	(0.0433)		
Observations	673	673	697	697
R-squared	0.552	0.779	0.471	0.684
Country FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Series are HP filtered.

Table 7: HOUSE PRICES, UNEMPLOYMENT AND PUBLIC SAVING

Table A	(1) Unemployment (2sls)	(2) Investment (2sls)	(3) Unemployment (2sls)
House Prices	-35.01***	35.85***	
	(10.65)	(11.12)	
Investment			-0.976***
			(0.157)
Observations	523	523	523

Table B	(1) Unemployment (OLS)	(2) Public Saving (OLS)	(3) Public Spending (OLS)	(4) Public Revenue (OLS)
Investment	-0.486***			
	(0.0439)			
Unemployment		-0.661***	0.626***	-0.103*
		(0.0584)	(0.0823)	(0.0552)
Observations	523	523	523	523
R^2	0.528	0.617	0.562	0.148

Notes: HAC robust standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). Country fixed-effects included. *** p<0.01, ** p<0.05, * p<0.1. This table is extracted from a paper on house prices and unemployment we are working on.

Table 8: Falsification tests I

	(1)	(2)	(3)	(4)	(5)	(6)
	Table A: Second-stage of the Instrumental Variable Strategy					
	CA	CA	CA	CA	CA	CA
First-stage Tax	Property	Income	Social Secu.	Payroll	Goods/Services	Other
House Prices	-23.15***	52.37	21.59	-176	-1.558	1.278
	(8.647)	(51.76)	(18.39)	(331.1)	(56.927)	(11.89)
Observations	769	769	769	769	769	769
	Table B: First-stage of the Instrumental Variable Strategy					
	House	House	House	House	House	House
Tax	Property	Income	Social Secu.	Payroll	Goods/Services	Other
	-0.0196***	0.00279	-0.00582*	-0.00241	7.41e-05	-0.0156**
	(0.00716)	(0.00207)	(0.00299)	(0.00488)	(0.00274)	(0.00758)
Observations	769	769	769	769	769	769

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and Year-fixed effects are included.

Table 9: Falsification tests II

	(1)	(2)	(3)	(4)
	House	House	House	House
	(2sls) 1st stage	(2sls) 1st stage	(2sls) 1st stage	(2sls) 1st stage
Tax	Property	Capital gains	Transactions	Inheritances
	-0.0184***	0.0484***	0.0696***	0.0167
	(0.00701)	(0.00913)	(0.00939)	(0.0254)
Observations	734	734	734	734

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and Year-fixed effects are included.

Table 10: Examining exclusion restriction

	(1)	(2)	(3)	(4)
	Property (/total tax)	Property (/GDP)	Property (/GDP)	Total tax (/GDP)
	(OLS)	(OLS)	(OLS)	(OLS)
GDP	-0.00285	-0.000419		
	(0.00187)	(0.000572)		
Property (/total tax)			0.286***	-0.555***
			(0.0167)	(0.170)
Observations	757	757	757	757
R^2	0.008	0.005	0.785	0.037

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included. Series are HP filtered. "Property" denotes the property tax.

Table 11: CONTROLLING BY DIFFERENT MEASURES OF GDP

	(1) House (IV: 1st st.)	(2) House (IV: 1st st.)	(3) House (IV: 1st st.)	(4) House (IV: 1st st.)	(5) CA (IV: 2nd st.)	(6) CA (IV: 2nd st.)	(7) CA (IV: 2nd st.)	(8) CA (IV: 2nd st.)
Property tax	-3.038***	-3.219***	-3.474***	-1.954**				
	(0.917)	(0.981)	(1.047)	(0.821)				
Relative income	1.547***				9.698			
	(0.201)				(12.07)			
GDP		0.00353***				0.00457		
		(0.000434)				(0.0241)		
GDP growth			0.0378**				0.701	
			(0.0180)				(0.457)	
GDP per cap.				4.34e-05***				0.000630
				(5.95e-06)				(0.000556)
House Prices					-20.82***	-19.66***	-20.95***	-26.45**
					(6.482)	(5.898)	(5.608)	(11.24)
Observations	665	593	592	593	593	593	592	593

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. Series are HP-filtered (except GDP growth).

Table 12: Share prices and current accounts?

	(1) CA (OLS)	(2) CA (OLS)	(3) Share Prices (OLS)	(4) CA (OLS)
Market cap.	0.00752			
	(0.0102)			
Share prices		-0.0216*		
		(0.0118)		
House Prices			36.05***	
			(8.300)	
Res. Share				-0.0101
				(0.0113)
Observations	517	517	517	517

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and Year-fixed effects are included. "Market cap." is market capitalization.

Table 13: Other HP filters

	(1) CA	(2) CA	(3) CA	(4) CA	(5) CA
Smooth. parameter	1600	400	100	25	10
	Table A: OLS				
	(OLS)	(OLS)	(OLS)	(OLS)	(OLS)
House Prices	-9.336***	-10.62***	-11.26***	-11.59***	-11.19***
	(0.937)	(1.068)	(1.236)	(1.478)	(1.648)
Observations	833	833	833	833	833
	Table B: IV				
	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)
House Prices	-20.19***	-17.10***	-14.97***	-13.48***	-13.04***
	(6.438)	(4.587)	(4.047)	(4.094)	(4.549)
Observations	769	769	769	769	769

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed-effects included.

Table 14: Other scaling variables: tax as a % of GDP, as a % of Investment and as a % of consumption

	(1) House prices (1st st.)	(2) House prices (1st st.)	(3) House prices (1st st.)	(4) CA (IV: % GDP)	(5) CA (IV: % Invest.)	(6) CA (IV: % Cons.)
Property tax (/GDP)	-0.0910***					
	(0.0264)					
Property tax (/Invest.)		-2.713***				
		(0.404)				
Property tax (/Cons.)			-6.224***			
			(1.702)			
House Prices				-18.87***	-24.75***	-25.78***
				(5.490)	(3.317)	(5.474)
Observations	766	766	766	766	766	766

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country-fixed effects are included. House Prices are an indice of house prices, normalized at 1 in 2005. CA denotes Current Account. All series are HP-filtered. The property tax variable is measured as a % of GDP, as a % of investment and as a % of consumption.

Table 15: Country Groupings

	(1) CA (OLS)	(2) CA (OLS)	(3) CA (OLS)	(4) CA (OLS)	(5) CA (OLS)
House Prices	-7.734***	-5.426**	-10.29***	-5.283***	-11.35***
	(1.657)	(2.190)	(2.579)	(1.475)	(2.617)
Public Surplus	0.0599	0.0803	0.0892	0.0600	0.0615
	(0.0884)	(0.0817)	(0.106)	(0.0629)	(0.161)
Relative income	0.912	-1.087	2.536	-16.71**	4.480
	(10.18)	(7.028)	(13.77)	(7.597)	(16.66)
Relative income sq.	-35.78	-643.2***	232.2	-47.70	-190.0
	(228.6)	(194.8)	(258.4)	(257.6)	(273.8)
Relative dependency ratio (Young)	-0.585**	-0.695**	-0.450	-0.710***	-0.976
	(0.244)	(0.275)	(0.357)	(0.202)	(0.617)
Relative dependency ratio (Old)	0.792*	1.328***	0.268	0.681	1.149*
	(0.451)	(0.458)	(0.847)	(0.558)	(0.634)
Financial deepening	0.0107	0.0227***	0.00113	0.0179**	0.00754
	(0.00831)	(0.00601)	(0.0124)	(0.00744)	(0.0127)
Oil Dummy	0.267		0.0153	0.278	1.381
	(1.059)		(1.838)	(0.951)	(1.038)
Real interest rates	0.109	-0.199	0.396**	0.115	0.275
	(0.170)	(0.169)	(0.190)	(0.152)	(0.348)
Real exchange rates	-0.0381***	-0.0643*	-0.0462***	-0.0434**	-0.0622**
	(0.0138)	(0.0331)	(0.0152)	(0.0169)	(0.0289)
Observations	402	170	232	201	201
R^2	0.254	0.533	0.265	0.524	0.309
Country FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Euro Countries		Yes	No		
High income Countries					Yes
Low income Countries				Yes	

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and Year-fixed effects are included. Series are detrended with a HP-filter. In our sample, Euro countries are Austria, Belgium, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, the Netherlands, Slovakia, Slovenia, Spain. High (low) income Countries are countries where GDP per capita is higher (lower) than the median of the sample.

Table 16: Consumer and firm credit constraints: more specifications

Table A	(1) CA (OLS)	(2) CA (OLS)	(3) CA (OLS)	(4) CA (OLS)	(5) CA (OLS)	(6) CA (OLS)	(7) Private Saving (OLS)	(8) Private Saving (OLS)	(9) Consumption (OLS)	(10) Consumption (OLS)	(11) CA (2sls)	(12) CA (2sls)
House Prices	-12.03***	-13.93***	-11.55***	-18.21**	-12.07***	-11.18***	-6.066***	-6.358***	14.42***	18.73***	-17.20**	-101.9
	(1.239)	(1.999)	(1.528)	(7.571)	(1.560)	(1.684)	(1.273)	(1.289)	(2.132)	(2.544)	(8.702)	(314.3)
House*LTV				0.0666								
				(0.0860)								
Observations	833	416	417	604	500	333	367	273	384	409	416	353
R^2	0.258	0.261	0.340	0.280	0.365	0.281	0.277	0.436	0.469	0.492		
LTV		$< 80\%$	$> 80\%$				$< 80\%$	$> 80\%$	$< 80\%$	$> 80\%$	$< 80\%$	$> 80\%$
Extraction					No	Yes						

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country and year fixed-effects are included. "Consumption" is indexed 2005=100 and in real terms.

Table B	(1) CA (OLS)	(2) CA (OLS)	(3) Invest. (OLS)	(4) Invest. (OLS)	(5) R Invest. (OLS)	(6) R Invest. (OLS)	(7) NR Invest. (OLS)	(8) NR Invest. (OLS)	(9) CA (2sls)	(10) Invest. (2sls)	(11) R Invest. (2sls)	(12) NR Invest. (2sls)
House Prices	-0.371	-1.593	7.983***	7.017***	5.579***	5.103***	2.370	1.935				
	(2.089)	(2.129)	(2.554)	(2.080)	(1.379)	(1.313)	(2.175)	(1.906)				
House/PCGDP	-938.7***	-931.6***	547.8**	581.6***	-7.756	50.76	556.6***	529.3***	-1,913***	3,513***	123.9	3,371***
	(209.3)	(191.1)	(218.0)	(190.4)	(87.33)	(86.90)	(187.4)	(174.9)	(710.3)	(879.9)	(308.1)	(928.0)
Observations	710	710	664	664	664	664	664	664	664	664	664	664
R^2	0.201	0.266	0.318	0.477	0.327	0.429	0.174	0.354				
Country FE	No	Yes	No	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1.

Table 17: Decomposition of the current account with Year Fixed Effects

	(1) OLS	(2) OLS,Y	(3) OLS	(4) OLS,Y	(5) OLS	(6) OLS,Y	(7) IV	(8) IV,Y	(9) IV	(10) IV,Y	(11) IV	(12) IV,Y
	Table A: Current Account = Savings - Investment											
	CA	CA	Saving	Saving	Invest.	Invest.	CA	CA	Saving	Saving	Invest.	Invest.
House Prices	-10.35***	-12.07***	2.618***	1.613	14.07***	13.80***	-17.66***	-24.70***	17.78***	16.32*	38.29***	42.17***
	(1.136)	(1.397)	(0.953)	(1.014)	(1.154)	(1.104)	(5.075)	(8.840)	(6.195)	(8.377)	(7.543)	(10.40)
Observations	721	721	721	721	721	721	721	721	721	721	721	721
R^2	0.156	0.230	0.020	0.239	0.305	0.475						
	Table B: Savings = Private Savings + Public Savings											
	Saving	Saving	Pr. Sav.	Pr. Sav.	Pu. Sav.	Pu. Sav.	Saving	Saving	Pr. Sav.	Pr. Sav.	Pu. Sav.	Pu. Sav.
House Prices	2.203***	1.187	-5.321***	-6.675***	8.375***	8.839***	27.16***	33.21*	-17.01***	-13.07	46.38***	47.26**
	(0.855)	(0.928)	(0.925)	(0.877)	(1.332)	(1.072)	(9.791)	(18.90)	(6.383)	(12.57)	(14.38)	(24.05)
Observations	621	621	621	621	621	621	621	621	621	621	621	621
R^2	0.016	0.271	0.073	0.298	0.113	0.527						
	Table C: Investment = Residential + Non-residential Investment											
	Invest.	Invest.	R Invest.	R Invest.	NR Invest.	NR Invest.	Invest.	Invest.	R Invest.	R Invest.	NR Invest.	NR Invest.
House Prices	11.49***	11.13***	4.605***	4.681***	6.829***	6.399***	33.68***	38.26**	1.475	-4.443	32.17***	42.17**
	(1.046)	(1.056)	(0.378)	(0.361)	(0.923)	(0.944)	(9.154)	(16.29)	(2.825)	(8.143)	(9.417)	(21.06)
Observations	591	591	591	591	591	591	591	591	591	591	591	591
R^2	0.273	0.468	0.365	0.467	0.134	0.387						

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects included in the regressions. Year fixed-effects included for OLS, Y and IV, Y columns. All series are detrended using a HP-filter.

Table 18: OLS Regressions with controls and year fixed effects

	(1) CA	(2) CA,Y	(3) CA	(4) CA,Y	(5) CA	(6) CA,Y	(7) CA	(8) CA,Y	(9) CA	(10) CA,Y
House Prices	-10.61***	-12.03***			-8.986***	-10.27***			-7.453***	-7.728***
	(1.068)	(1.239)			(1.396)	(1.815)			(1.225)	(1.657)
Public Surplus			-0.106*	-0.000113	-0.0295	0.0557	-0.0516	0.0273	0.00266	0.0591
			(0.0622)	(0.0862)	(0.0580)	(0.0837)	(0.0733)	(0.0901)	(0.0704)	(0.0885)
Relative income			-8.018	-12.73	1.861	0.855	-4.878	-8.489	2.271	0.940
			(8.111)	(9.324)	(7.849)	(9.074)	(8.810)	(9.924)	(8.915)	(10.18)
Relative income sq.			-70.92	4.282	-9.645	39.57	-227.0	-110.4	-114.7	-36.34
			(201.1)	(217.0)	(178.7)	(186.4)	(262.0)	(253.2)	(239.1)	(228.5)
Relative dependency ratio (Young)			-0.247	-0.274	-0.441*	-0.371	-0.430*	-0.494**	-0.696***	-0.583**
			(0.253)	(0.262)	(0.240)	(0.250)	(0.260)	(0.251)	(0.248)	(0.244)
Relative dependency ratio (Old)			0.365	0.569	0.0109	0.141	1.003**	1.390***	0.446	0.796*
			(0.444)	(0.465)	(0.434)	(0.446)	(0.471)	(0.455)	(0.468)	(0.451)
Financial deepening			0.00731	0.00642	0.00954	0.00845	0.00673	0.0104	0.00830	0.0107
			(0.00659)	(0.00999)	(0.00589)	(0.00909)	(0.00606)	(0.00843)	(0.00574)	(0.00830)
Oil Dummy			-0.359	-0.490	-0.510	-0.602	0.218	0.0349	-0.0789	-0.206
			(0.828)	(0.831)	(0.787)	(0.761)	(0.805)	(0.776)	(0.782)	(0.755)
Real interest rates							0.136	0.138	0.190	0.105
							(0.161)	(0.173)	(0.154)	(0.171)
Real exchange rates							-0.0572***	-0.0570***	-0.0329**	-0.0380***
							(0.0177)	(0.0153)	(0.0153)	(0.0138)
Observations	833	833	473	473	473	473	402	402	402	402
R^2	0.181	0.258	0.044	0.122	0.170	0.232	0.086	0.194	0.168	0.254

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects included in the regressions. Year fixed-effects included in columns CA,Y.

Table 19: Instrumental strategy with controls and year fixed effects, First Stage

	(1) House (2sls)	(2) House (2sls)	(3) House (2sls)	(4) House (2sls)	(5) House (2sls)	(6) House (2sls)	(7) House (2sls)	(8) House (2sls)	(9) House (2sls)	(10) House (2sls)
Property tax	-3.697***	-1.956***	-3.389***	-1.800**	-3.362***	-1.732**	-3.195***	-1.658**	-2.955***	-1.560**
	(0.881)	(0.716)	(0.942)	(0.787)	(0.934)	(0.757)	(0.971)	(0.778)	(0.951)	(0.793)
Rel. dependency ratio (young)			-0.0318***	-0.0234***	-0.0299***	-0.0207***	-0.0282***	-0.0223***	-0.0299***	-0.0233***
			(0.00728)	(0.00630)	(0.00772)	(0.00686)	(0.00714)	(0.00659)	(0.00759)	(0.00675)
Rel. dependency ratio (old)			-0.0112	-0.0283**	-0.00890	-0.0239*	-0.0122	-0.0234	-0.0153	-0.0319**
			(0.0157)	(0.0141)	(0.0155)	(0.0138)	(0.0156)	(0.0148)	(0.0160)	(0.0144)
Oil Dummy			0.00248	0.0228	-0.0255	-0.00828	0.00249	0.0197	-0.0439	0.0111
			(0.0288)	(0.0202)	(0.0326)	(0.0269)	(0.0292)	(0.0193)	(0.0398)	(0.0716)
Relative income					0.181***	0.166***				
					(0.0639)	(0.0596)				
Relative income sq.					3.133	2.398				
					(7.794)	(6.714)				
Real exchange rates							0.00345***	0.00268***		
							(0.000700)	(0.000599)		
Financial Deepening									0.00182***	0.00115***
									(0.000405)	(0.000355)
Observations	769	769	591	599	591	599	567	575	545	553
Year FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects included in the regressions. All series are HP-filtered.

Table 20: Instrumental strategy with controls and year fixed effects, Second Stage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	CA	CA	CA	CA	CA	CA	CA	CA	CA	CA
	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)
House Prices	-17.10***	-23.15***	-18.77***	-26.89***	-19.09***	-27.82***	-22.25***	-32.48***	-22.62***	-34.35***
	(4.587)	(8.647)	(5.438)	(10.24)	(5.417)	(10.50)	(6.070)	(12.20)	(6.572)	(13.09)
Relative dependency ratio (young)			-0.292	-0.359	-0.277	-0.321	-0.438*	-0.569	-0.529**	-0.606
			(0.241)	(0.317)	(0.233)	(0.305)	(0.247)	(0.359)	(0.254)	(0.384)
Relative dependency ratio (old)			-0.205	-0.413	-0.183	-0.360	-0.378	-0.681	-0.0759	-0.577
			(0.325)	(0.488)	(0.327)	(0.472)	(0.362)	(0.525)	(0.370)	(0.617)
Oil Dummy			0.355	0.989	0.168	0.493	0.364	1.066	-1.784	-0.399
			(0.912)	(0.871)	(0.998)	(1.000)	(0.956)	(0.910)	(1.713)	(2.453)
Relative income					2.237	3.315				
					(2.480)	(2.944)				
Relative income sq.					-153.3	-65.61				
					(203.1)	(220.7)				
Real exchange rates							0.0206	0.0316		
							(0.0265)	(0.0390)		
Financial Deepening									-0.0188	-0.0182
									(0.0175)	(0.0197)
Observations	769	769	591	599	591	599	567	575	545	553
Year FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Country fixed effects included in the regressions. All series are HP-filtered.

Table 21: Instrumental strategy with controls, Investment, Second Stage

	(1) Inv. (2sls)	(2) Inv. (2sls)	(3) Inv. (2sls)	(4) Inv. (2sls)	(5) Inv. (2sls)	(6) Inv. (2sls)	(7) Inv. (2sls)	(8) Inv. (2sls)	(9) Inv. (2sls)	(10) Inv. (2sls)
House Prices	38.29***	42.17***	39.37***	46.95***	39.23***	47.69***	45.77***	58.12***	45.96***	58.88***
	(7.541)	(10.40)	(8.712)	(14.57)	(8.762)	(15.06)	(10.31)	(19.34)	(11.53)	(22.14)
Relative dependency ratio (young)			0.370	0.223	0.335	0.183	0.548	0.604	0.669	0.660
			(0.351)	(0.430)	(0.339)	(0.410)	(0.393)	(0.555)	(0.436)	(0.619)
Relative dependency ratio (old)			0.0603	1.001	0.0169	0.910	0.286	1.389*	0.268	1.563
			(0.503)	(0.673)	(0.495)	(0.647)	(0.590)	(0.833)	(0.636)	(1.013)
Oil Dummy			-0.239	-0.269	-0.170	-0.387	-0.264	-0.312	0.827	0.530
			(0.919)	(0.887)	(0.954)	(0.972)	(1.135)	(1.155)	(1.157)	(1.434)
Relative income					-3.116	-3.800				
					(2.559)	(3.525)				
Relative income sq.					-290.0	-172.9				
					(211.5)	(209.9)				
Real exchange rates							-0.121***	-0.105*		
							(0.0435)	(0.0553)		
Financial Deepening									-0.0521**	-0.0416
									(0.0255)	(0.0309)
Observations	721	721	590	590	590	590	566	566	544	544
Year FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. All series are HP-filtered.

Table 22: Instrumental strategy with controls, Saving, Second Stage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Sav.	Sav.	Sav.	Sav.	Sav.	Sav.	Sav.	Sav.	Sav.	Sav.
	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)	(2sls)
House Prices	17.78***	16.32*	17.98**	16.30	17.66**	15.76	21.49***	22.25*	23.73***	25.97*
	(6.194)	(8.377)	(7.087)	(10.62)	(7.030)	(10.98)	(7.677)	(12.62)	(8.691)	(14.44)
Relative dependency ratio (young)			0.0852	-0.147	0.0810	-0.131	0.120	-0.00290	0.224	0.133
			(0.285)	(0.303)	(0.273)	(0.288)	(0.284)	(0.347)	(0.320)	(0.387)
Relative dependency ratio (old)			-0.469	0.0628	-0.471	0.0745	-0.437	0.0899	-0.0810	0.683
			(0.337)	(0.453)	(0.326)	(0.423)	(0.358)	(0.499)	(0.416)	(0.616)
Oil Dummy			0.184	0.191	0.492	0.419	0.170	0.167	-0.433	-0.695
			(0.675)	(0.633)	(0.700)	(0.662)	(0.781)	(0.719)	(0.868)	(0.928)
Relative income					0.367	1.423				
					(2.184)	(2.623)				
Relative income sq.					-253.0	-101.9				
					(163.7)	(132.0)				
Real exchange rates							-0.118***	-0.0942***		
							(0.0322)	(0.0347)		
Financial Deepening									-0.0596***	-0.0475**
									(0.0207)	(0.0213)
Observations	721	721	590	590	590	590	566	566	544	544
Year FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. All series are HP-filtered.

Table 23: CONTROLLING WITH THE FREQUENCY OF CADASTRAL REVISIONS

	(1) House (IV: 1st st.)	(2) House (IV: 1st st.)	(3) House (IV: 1st st.)	(4) CA (IV: 2nd st.)	(5) CA (IV: 2nd st.)	(6) CA (IV: 2nd st.)
Property tax	-3.697***	-3.434***	-3.896***			
	(0.881)	(1.193)	(1.281)			
House Prices				-17.10***	-17.37***	-16.92***
				(4.588)	(6.404)	(6.366)
Observations	769	393	376	769	393	376
Revision		Yes	No		Yes	No

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. All series are HP-filtered. "Revision" points to countries where cadastral values are reassessed at least every five years (Australia, Canada, Denmark, Finland, Hungary, Japan, Netherlands, New Zealand, Portugal, South Africa, Sweden, Switzerland, United States). For a description of the frequency of revision of cadastral values, see Table 26.

Table 24: DECADES

	(1) House (IV: 1st st.)	(2) House (IV: 1st st.)	(3) House (IV: 1st st.)	(4) CA (IV: 2nd st.)	(5) CA (IV: 2nd st.)	(6) CA (IV: 2nd st.)
Property tax	-2.473**	-3.705***	-10.68***			
	(0.987)	(1.281)	(2.568)			
House Prices				-20.47*	-20.58***	-13.60***
				(11.90)	(6.647)	(4.491)
Observations	284	229	256	284	229	256
Decades	<1990	1990s	>2000	<1990	1990s	>2000

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. All series are HP-filtered.

Table 25: Real Exchange Rates

	(1) CA (IV)	(2) Real Exchange Rates (OLS)	(3) Real Exchange Rates (VAR)	(4) House (VAR)
House Prices	-19.73***	18.83***		
	(5.017)	(3.532)		
Real exchange rates	0.0293			
	(0.0189)			
Real Exchange Rates (L1)			0.461***	-0.000404
			(0.0723)	(0.000341)
House Prices (L1)			11.33***	0.707***
			(3.155)	(0.0623)
Observations	691	691	664	664
R^2		0.052	0.287	0.447

Notes: HAC robust (Heteroscedasticity and AutoCorrelation robust) standard errors are in parentheses (we use Bartlett kernel-based filter with bandpass parameter 2). *** p<0.01, ** p<0.05, * p<0.1. Country fixed effects are included in the regressions. All series are HP-filtered.

1 Data Appendix

Instrumental variable. We use a real-estate property tax variable. We focus on recurrent taxes as real estate capital gain taxes (non-recurrent tax) are endogenously affected by house prices. Real-estate includes land, buildings, and other construction or « improvements » to land. Precise categorization of taxpayers is difficult in practice. We thus include both property taxes and dwellings taxes in the variable [25]. Data are built from the taxation series of OECD.

Frequency of revisions of the cadastral values. See Table 26.

Data on LTV ratios. As it is common in the literature (see: Andrews and Caldera Sanchez (2011) or Andrews (2011)), we use the maximum LTV ratios. The maximum LTV accounts for the maximum access to financing that the mortgage market grants to households. High LTV ratios are associated with low downpayment requirements. Data are sourced from Chiuri and Jappelli (2003), Jappelli and Pagano (1994), Catte et al. (2004), Green and Wachter (2005), ECB(2009), Andrews et al. (2011).

Home equity extraction dummy. Home equity extraction exists when households may take up debt secured on the housing stock and use it for consumption spending. There are 9 countries in our sample where home equity extraction exists (Australia, Canada, Denmark, Finland, the Netherlands, Norway, Sweden, the United Kingdom, the United States). For a description see in particular table in Appendix 1 "Mortgage equity withdrawal" in Andrews (2010). Mortgage equity withdrawal appears to be less common in euro area housing markets (ECB 2009 ; Catte et al. 2004).

Data on house prices. See Table 27.

[25] For example in France, this variable is mainly composed of the "taxe d'habitation" (dwelling tax) and "taxe foncière" (property tax). Dwelling taxes also affect house prices as they reduce the (before-tax) rental income earned by owners, and so house prices.

Table 26: Recurrent Taxes on Property: Revision of Cadastral Values

Country	Level of administration of the Tax	Cadastral values	Sources
Australia	Local councils levy rates on the rental value of the property	Land valuations made every 3 to 4 years	Landgate (2012), Sidney (2012)
Austria	Federal rate multiplied by a municipal coefficient	From 1973 with no automatic update	ECB (2012)
Belgium	Regional and Local	From 1975, indexed to the CPI since 1991	ECB (2012)
Canada	Municipal governments	Market value in most provinces (with an annual reassessment)	Statistics Canada (2003)
China	Central, local	On historical cost. Market value for Shanghai and Chongqing since 2011	The Economist (2012)
Czech Republic	Local	Based upon floor-area	ECB (2012)
Denmark	Municipal tax and National tax	Updated every second year	ECB (2012)
Estonia	Municipality	From 2001	ECB (2012)
Finland	Municipality	From 2009	ECB (2012)
France	Local	From 1978	Sénat (2012)
Germany	Federal rate multiplied by a municipal coefficient	From 1964	ECB (2012)
Greece	National tax of 2011	Based upon floor-area	ECB (2012)
Hungary	Local	Fair market value	ECB (2012)
Ireland	National	Regular update for non-residential housing. New Property Tax in 2012	ECB (2012)
Italy	Local Tax	From 1988. Correction factor was increased by 60% in 2012	ECB (2012)
Korea	Local and national	From 2005	Kim (2008)
Japan	Central government	Adjusted every three years	The Japan Times (2012)
Luxemburg	Local	From 1941	ECB (2012)
Netherlands	Local	Updated annually by municipalities	ECB (2012)
New Zealand	Local	Official land valuation every three years	LINZ (2012)
Norway	Municipalities	Assessed value of the property (about 25% of the market value)	Global Property Guide (2012)
Portugal	Municipalities (min/max rates determined at the national level)	Adjusted every 3rd year. But some values have not been updated since 2003	ECB (2012)
Slovak Republic	National and municipalities	From 2004	ECB (2012)
Slovenia	Municipalities	Based upon floor-area. Market value since 2012.	ECB (2012)
South Africa	Local	Market value	Global Property Guide (2012)
Spain	Tax levied by municipalities	Partly updated in Jan. 1994	ECB (2012)
Sweden	Municipal tax	Fully updated every 6th year, with a minor revision in between	ECB (2012)
Switzerland	Cantons	Market value	Federal Tax Administration (2011)
United Kingdom	Local taxation (Council tax)	From April 1991	ECB (2012)
United States	Local governement level (municipal or county level)	Nearly always at the fair market value. Values determined by local officials	Texas Basics, Tax Foundation Study

Table 27: Data Appendix: House Price Series

Country	Time coverage	Sources	Series
Australia	1970-present	BIS -Australian Treasury	Residential property prices, existing dwellings (8 Cities), nsa
Austria	1986-present	BIS	Residential Property Prices, all dwellings (Vienna and big cities), nsa
Belgium	1970-present	BIS-Statistics Belgium	Residential property prices, existing houses, nsa.
Canada	1970-present	BIS	Residential Property Prices, all dwellings
China	1998-present	BIS	Land prices, residential and commercial, nsa
Czech Republic	2004-present	BIS	Residential property prices, existing flats , nsa
Denmark	1970-present	Danmarks Nationalbank	Residential property prices, new and existing single-family house, nsa
Estonia	2002-present	BIS	Residential property prices, all flats, nsa
Finland	1970-present	BIS-Statistics Finland	Residential property prices, existing houses, nsa
France	1970-present	J. Friggit (Conseil Général à l'Environnement et au Développement Durable)	Residential property prices, existing dwellings, nsa
Germany	1975-present	BIS- Deutsche Bundesbank	Residential property prices, existing flats (West-G.), nsa
Greece	1992-present	BIS	Residential property prices, all flats (Athens-Thessaloniki), nsa
Hungary	2000-present	BIS	Residential property prices, existing dwellings (Budapest), nsa
Iceland	1999-present	BIS	Residential property prices, all dwellongs, nsa
Indonesia	2001-present	BIS	Residential property prices, new houses (big cities), nsa
Ireland	1970-present	BIS-Department of Environment	Residential Property Prices, all dwellings, nsa
Israel	2000-present	BIS	Residential property prices, owner-occupied dwellings, nsa
Italy	1970-present	BIS- Il Consulente Immobiliare	Residential Property prices, All dwellings, nsa.
Japan	1970-present	Stat Bureau, Ministry of Internal Affairs and Communications, Japan	Japan Residential land price index
Korea	1985-present	BIS	Residential Property Prices, all dwellings, nsa
Mexico	2004-present	BIS	Residential Property Prices, all dwellings, nsa
Netherlands	1975-present	BIS-The Dutch Land Registry Office (Kadaster)	Residential Property Prices, existing dwellings, nsa
New Zealand	1970-present	BIS-Reserve Bank of New Zealand	Residential Property Prices, all dwellings, nsa
Norway	1970-present	Norges Bank	Residential Property Prices, all dwellings, nsa
Poland	2001-present	BIS	Residential property prices, existing flats (big cities), nsa
Portugal	1987-present	BIS	Residential Property Prices, all dwellings, nsa
Slovak Republic	2004-present	BIS	Residential Property Prices, existing dwellings, nsa
Slovenia	2002-present	BIS	Residential Property Prices, existing dwellings, nsa
South Africa	1970-present	BIS-ABSA	Residential Property Prices, all middle-segment houses, nsa
Spain	1970-present	BIS-Ministerio de la Vivienda	Residential Property Prices, all dwellings, nsa
Sweden	1970-present	BIS-Statistics Sweden	Residential Property Prices, all owner-occupied dwellings, nsa
Switzerland	1970-present	Swiss National Bank	Residential Property Prices, all 1-family houses, nsa
United Kingdom	1970-present	Nationwide	Residential Property Prices, all dwellings, nsa
United States	1970-present	FHFA-Shiller	Residential Property Prices, existing 1-family houses, nsa

Variables	Abbreviation	Sources	Variable description
House Prices	House	See Table 27	Real house prices (base 100=2005)
Current account balance	CA	WDI	Current account balance (ratio of GDP)
Property Tax	Property Tax	OECD	Property tax (ratio of total taxation)
Income Taxes	Income Tax	OECD	Income tax (ratio of total taxation)
Taxes on capital gains	Capital gains	OECD	Taxes on capital gains (ratio of total taxation)
Taxes on inheritances	Inheritances	OECD	Taxes on inheritances (ratio of total taxation
Taxes on capital and financial transactions	Transactions	OECD	Taxes on capital and financial transactions (ratio of total taxation)
Social security contributions	Social Secu.	OECD	Social security contributions (ratio of total taxation)
Payroll taxes	Payroll	OECD	Payroll taxes (ratio of total taxation)
Taxes on goods and services	Goods/Services	OECD	Taxes on goods and services (ratio of total taxation)
Other taxes	Other	OECD	Other taxes (ratio of total taxation)
CPI	CPI	OECD	Consumer Prices, Index 2005=100
Net Foreign Asset Position	NFA	WDI	Stock of net foreign assets, ratio to GDP
Young dependency ratio	Relative dependency ratio (Young)	WDI	Youth Population under 15/Population between 15 and 65
Old dependency ratio	Relative dependency ratio (Old)	WDI	Population over 65/Population between 15 and 65
Gross fixed capital Formation	Investment	OECD	Gross fixed capital Formation, total, ratio of GDP
Residential Investment	Res. Inv.	OECD	Gross fixed capital formation (housing), ratio of GDP
Non residential Investment	NR Invest.	OECD	Gross fixed capital formation (non-housing), ratio of GDP
Saving	Gross domestic savings	WDI	Gross domestic savings (ratio of GDP)
Gross Saving	Gross Savings	WDI	Gross savings (ratio of GDP)
Government net lending	Government surplus	OECD	Government net lending (+ indicates surplus, - indicates deficit), ratio of GDP
Net Capital Outlays	Net Capital Outlays	OECD	Net capital outlays of the government, ratio of GDP
Public Saving	Public Saving	OECD	Government net lending+ Net capital outlays, ratio of GDP
Private Saving	Private Saving	OECD	Gross Savings minus Government net lending minus Net capital outlays , ratio of GDP
Total General government expenditure	Public Spending	OECD	Total General government expenditure, ratio of GDP
Total General government revenue	Public Revenue	OECD	Total General government revenue, ratio of GDP
Household final consumption	Consumption	WDI	Household final consumption expenditure, etc. (ratio of GDP)
Share Prices	Share Prices	OECD	Share prices, Index 2005 = 100
Relative Income	Relative Income	WDI	Relative income is the the GDP per capita divided by the GDP per capita for the US
GDP	GDP	WDI	GDP (current US $), Index 2005=100
GDP per capita	GDP per capita	WDI	GDP per capita, PPP (current international $)
Market capitalization	Financial deepening	WDI	Market capitalization of listed companies (ratio of GDP)
Domestic credit to private sector	PCGDP, Financial. Deep.	WDI	Domestic credit to private sector (ratio of GDP)
Oil rents	Oil rents	WDI	Oil rents (ratio of GDP)
Oil dummy	Oil dummy		Norway, Russia
Real long term interest rates	Real Interest Rates	OECD	Real long-term interest rate on government bonds, ratio of GDP
Real effective exchange rate	Real exchange rates	WDI	Real effective exchange rate index (2005 = 100)
Unemployment	Unemployment	WDI	Unemployment, total (% of total labor force)
Loan-To-Value ratios	LTV ratios	See text	maximum LTV ratios

2 Stationarity

There are two types of test we can use to test the stationarity of the residuals. We can reject the joint hypothesis that all residuals contain a unit root quite easily using panel-data unit root tests. Results are displayed in Table 28. Because the sample must be a balanced panel in order to perform the existing panel test procedures, the sample is restricted to 30 years (1979 − 2009) and 15 countries. For example, the Levin-Lin-Chu bias-adjusted t statistic are significant at all the usual testing levels. Therefore, we reject the null hypothesis and conclude that these series are stationary. Note however that all these tests assume a common autoregressive parameter for all series, so this test does not allow for the possibility that some country residuals contain unit roots while other country residuals do not.

To test for whether just one country residuals contain a unit root is arguably harder to reject. However, for those countries for which we have a reasonable number of years (that is, higher than 10 years), we can reject the hypothesis that our series contain unit roots at the usual confidence levels (5%). Therefore, we do not have to worry about spurious regressions problems when using a smoothing parameter of 400.

Table 28: Test Statistics: Panel of series

	Residual
Levin-Lin-Chu	-12.2587***
Im-Pesaran-Shin	-4.2698***
Fisher-type tests (ADF)	68.3449***
Harris-Tzavalis	0.5663***

Notes: *** p<0.01, ** p<0.05, * p<0.1.

3 Comparability of house price indexes

It is important for the validity of our study that house price indexes be comparable across countries. Since we at times use prices in capital cities or commercial prices instead of plain residential countrywide house price indexes, we check that all prices are the same, be there residential or commercial, countrywide or limited to one big capital city. Intuitively, all this can be understood by an arbitrage argument: residential structures can be turned into office space and the reverse (and land prices joint determine both residential and commercial real estate prices), and residents in the country arbitrage between different cities. In contrast, there are very high costs to arbitrage between real estate markets of two different countries, such as language and culture, which drives a wedge between house prices in different countries.

4 The Spanish example: four policy shocks

We develop four exogenous tax shocks which happened in Spain during the last thirty years. We show that all of them are not related to current accounts. Information about the history of

Table 29: Test Statistics: Individual Tests for Residuals

	Obs.	Phillips-Perron	Dickey-Fuller
Australia	38	0.00133896	0.00057261
Austria	23	0.13036501	0.13946425
Belgium	7	*0.38942575*	*0.35585806*
Canada	34	3.87E-07	4.49E-07
Czech Republic	5	0.00964448	0.0295434
Denmark	34	0.00011012	0.00013367
Estonia	6	*0.79614642*	*0.62502458*
Finland	34	0.04314767	0.09078547
France	34	0.01521007	0.01261114
Germany	34	0.032882	0.03641409
Greece	16	0.18450038	0.20636683
Hungary	8	*0.6829401*	*0.77173235*
Iceland	9	*0.02059045*	*0.0204428*
Ireland	35	0.03074412	0.04414738
Israel	8	*0.28900171*	*0.21027767*
Italy	29	0.11187055	0.14829428
Japan	32	0.065605	0.10567722
Korea	23	0.02037664	0.02049362
Mexico	4	*0.45424782*	*0.45656888*
Netherlands	33	0.00970076	0.01244231
New Zealand	37	0.00021194	0.00025407
Norway	34	0.03045081	0.02397773
Poland	7	*0.35994073*	*0.32838974*
Portugal	21	0.01285676	0.01429522
Slovak Republic	4	*0.59535082*	*0.56657353*
Slovenia	6	*0.07901901*	*0.08319547*
Spain	34	0.03767954	0.05407641
Sweden	39	0.0011417	0.00111046
Switzerland	32	0.00365723	0.00272435
United Kingdom	39	0.0121957	0.01067992
United States	39	0.02724493	0.03492188

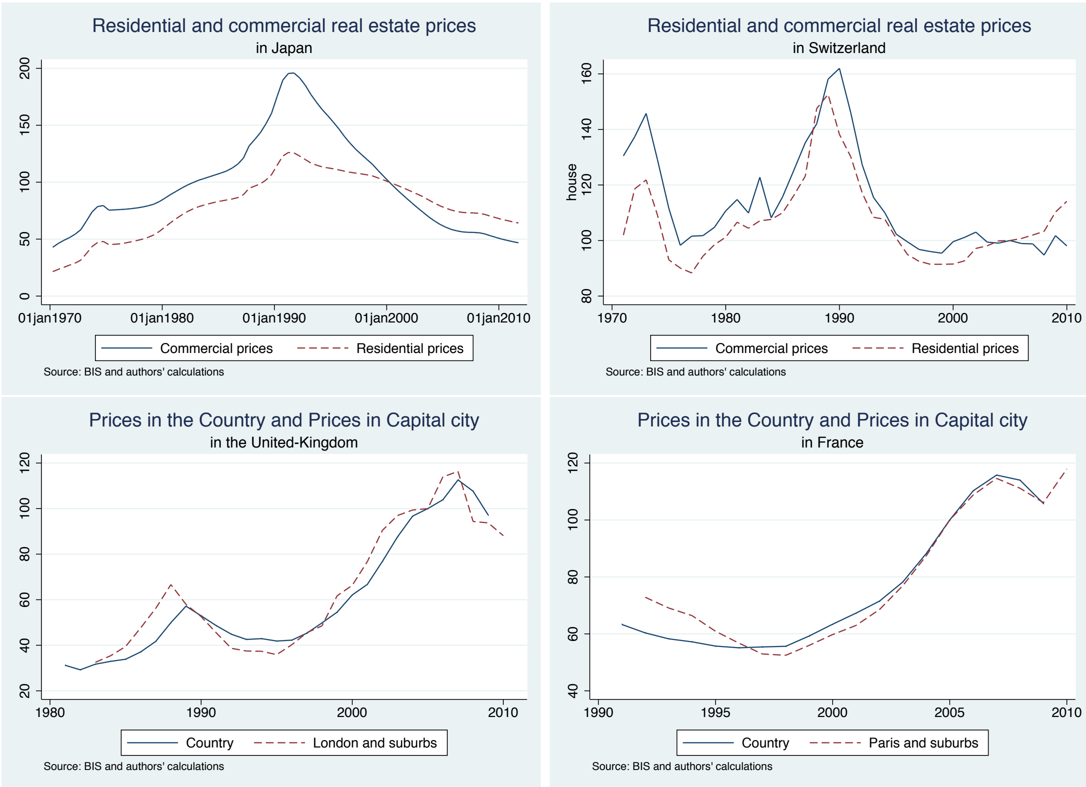
Notes: Augmented Dickey-Fuller and Phillips-Perron tests are reported for residuals of the regression of current-accounts on house prices with country and year-fixed effects.

property taxation in Spain are taken more specifically from Jesús Miranda Hita (The cadastre and real estate tax, 2004).

1. **1979-1980: Decree Law of 1979**

   In 1979, a decree Law (11/1979) introduced an extensive package of measures for the reorganization of local treasuries, ranging from doubling the base of some property taxes (the Urban Land Tax) and the subsequent revision of all cadastral values. Property taxes were also converted into local taxes. Property taxes were increased in a context of structural deficits of local communities. Indeed, social demands had increased since 1972 (the arrival of democracy) and were materialized with central government deficit. The government responded to those demands by exporting deficit to the local authorities. The package of measures provided in the decree law of 1979 addressed the "chronic situation of structural deficit of Local Corporations". With this perspective, the decree law proposed to adjust the taxable bases of land taxes and to update the cadastral value of urban estates. (The coefficients were different for each local corporation depending on which year the "cadastral system" of Law 41/1964 had been implemented.) The goal was the increase in cadastral income of rented housing and premises, the elimination of certain hypotheses established in Land Tax law, the annulment of certain exemptions and rebates and the reduction

Figure 4: Comparability of house prices indexes



Note: Because of data availability, we show 4 countries where data exist in BIS database. We restrict ourselves to data from BIS database to ensure data comparability so that covered area are strictly similar when we compare residential and commercial prices, or property types are strictly the same when we compare prices in the whole country and in the capital city.

in the amount of others. The movement was followed by the law of 1983 (see next shock).

2. **1985: the sentence of the Constitutional Court on 19 December 1985**

   In 1983, a law (24/1983) contained a package of measures designed to reinforce the capacity of local self-finance: it authorized local authorities to establish a surcharge on Personal Income Tax and on property taxation. The surcharge was effectively applied, amidst fierce debate, by 528 local corporations that year. But this surcharge was later overturned by sentence of the Constitutional Court on 19 December 1985. It resulted in a decrease in property taxation.

3. **1991-1994: the law of 1987, first applied after the local elections of 1991.**

   There was an increase the rate of land taxes, which from then on could for example vary between 20 and 40 percent for urban land [26]. But it is only in 1991, after

[26] Law of local treasuries of 1988 reinforced the autonomy of municipalities by authorizing Local Corporations to establish two additional taxes (Tax on the Increase in Value of Urban Land and a Tax on Construction, Installations and Works)

the municipal elections [27], that the municipalities increased the rates of land taxes. Between 1991 and 1993 local corporations showed a high level of activity, increasing rates annually from 0.588 in 1990 to 0.664 in 1993. This explains that the increase in property taxation was gradual in this period. This activity is largely attributable to the absence of cadastral value revisions in this three-year period. When revisions were resumed effective 1 January 1994, we observe that the average rate went down that year to 0.658, and continued a downward trend.

4. **2006 : the law of 2006**

   At the end of 2006, Spain got a new tax reform. More precisely, on 30th of November came Law 36/2006 with measures to prevent tax frauds. Through this law of November 29, the power to require the production of a cadastral declaration for new constructions was attributed to municipalities when granting the license authorizing the first occupation of buildings. Moreover, the Act modified the fiscal scheme, changing the regime for net taxable income for certain properties for the purposes of the local property tax. In practice, the law led to an increase of the local property tax (Impuesto de Bienes Inmuebles) [28].

[27] Municipal elections are every four year in Spain. So the first elections after 1987 were in 1991)

[28] This is a rate tax which varies depending on the municipality and the level of urbanisation and services relating to your property. It is based on the "valor catastral" (fiscal value), determined by the cadaster office in the provincial capitals. This yearly tax is 0,4% on residential property, but municipalities may increase this percentage in accordance with the number of inhabitants and the services given.

Chapter 1

Reassessing Dynamic Efficiency

Abstract

In a seminal paper, Abel et al. (1989) argue that the United States and 6 other major advanced economies are dynamically efficient. Updating data on mixed income and land rents, I find in contrast that the criterion for dynamic efficiency is not verified for any advanced economy ; and that Japan and South Korea have unambiguously over-accumulated capital. This world "savings glut" can potentially explain otherwise hard-to-understand macroeconomic stylized facts (low interest rates, cash holding by firms, financial bubbles). Subject to some caveats, an increase of public debt, or a generalization of pay-as-you-go systems could therefore be Pareto-improving.

Keywords: Dynamic efficiency, saving and investment, intergenerational income distribution.

JEL classification: E21, E22, E24

Acknowledgements: I especially thank Emmanuel Farhi, Gregory Mankiw, Lawrence Summers, Andrei Shleifer and Richard Zeckhauser for advice and helpful discussions. I thank Markus Brunnermeier, John Campbell, Arnaud Costinot, Christian Gollier, David Laibson, Owen Lamont, Guy Laroque, Philippe Martin, Jonathan Parker, Thomas Piketty, Thomas Philippon, Adriano Rampini, Erik Röder, Kenneth Rogoff, José Scheinkman, Hyun Song Shin, Jaume Ventura, Philippe Weil and seminar participants at Collège de France, Harvard University, INSEE-CREST, and Massachusetts Institute of Technology for comments and suggestions. All errors remain mine. I am extremely grateful to "Corps des Ponts" for funding this research and to Harvard University for their hospitality.

Introduction

Dynamic efficiency is an important macroeconomic issue. The presumption that increasing investment is always good for the economy relies on the fact that more investment leads to more output in the long run, and that more output helps achieve higher consumption. Government debt is similarly often criticized as having crowding-out effect on capital accumulation, because decreasing investment is assumed to be detrimental to economic growth. But this need not be the case. A competitive equilibrium with optimizing agents, market clearing, price taking and rational expectations can fail to be Pareto-optimal when interest rates are low in the competitive equilibrium, even without assuming any type of inefficiency (externalities, information asymmetries, or price stickiness): all that is needed is that the economy is expected to run forever, which is ruled out in the canonical Arrow-Debreu model.[1] Samuelson (1958) and Diamond (1965) showed that the government can in this case make every agent better off by borrowing.[2] [3]

For example, in a Diamond (1965) overlapping-generations capital accumulation model without uncertainty, the capital stock can be too large in the competitive steady-state: in that case, the interest rate r is lower than the rate of growth of the economy g. The economy is *dynamically inefficient*. To the contrary, the economy is dynamically efficient if the interest rate is high: $r > g$. When $r = g$, the economy is said to be at the Golden Rule level of capital accumulation.[4]. This result is not as counterintuitive as it might seem at first glance. Sure, more investment always leads to higher output. But if agents care about consumption, and not output, a social planner might realize that maintaining the capital/output ratio requires more in investment each year (gK) than the economy actually produces in capital income (rK). Intuitively, a capital sector that is on net producing more output than it is using for new investment is contributing to consumption, whereas one that is using more in resources than it is producing is a sink for scarce resources.

The idea that the economy has accumulated enough capital may seem counterintuitive, to say the least, to many. With an ageing population, shouldn't the old generation care more about the less numerous young generation and help reduce public debt, rather than the other way around? But the idea behind dynamic inefficiency (and in the OLG model) is that capital might actually do a poor job at transferring resources ; it might well be more efficient to have the young work for the old in each period (a pay-as-you-go system), rather than having every individual save for retirement through its own retirement account, because capital is not so efficient at producing fruits needed at retirement. More importantly, this analysis is not only normative but also positive. For if the economy is in a dynamically inefficient state then rational bubbles can appear[5], and transfer resources from young to old agents though

[1]Technically, the failure of the first welfare theorem relies on the double infinity of agents (new agents are born in each period $t = 1, 2, ..., \infty$) and goods (if the consumption good is perishable, then there is at least one good in each period $t = 1, 2, ..., \infty$), as explained in Shell (1971). In order to prove the first welfare theorem, one needs that the sum of endowment values are summable at the equilibrium price vector, and hence that interest rates are not too low. The first welfare theorem extends to cases in which there are an infinite number of agents or of goods, but not when there are both an infinite number of agents and goods. What is interesting is that the usual suspects for inefficiencies are here absent : there are no externalities, no information asymmetries, and all markets are assumed to be clearing (no price stickiness in particular). As Geanakoplos (2008) remarks: "On the whole, it seems at least as realistic to suppose that everyone believes the world is immortal as to suppose that everyone believes in a definite date by which it will end. (In fact, it is enough that people believe, for every T, that there is positive probability the world lasts past T.)"

[2]The government can just as well run other types of Ponzi-schemes: social security, money, etc.

[3]Of course, unbacked public debt is akin to a rational bubble, and its uncertain refinancing can introduce additional uncertainty in the economy. But the government can introduce a broad spectrum of measures to ensure coordination on this rational bubble rather than on others, ranging from accepting Treasuries in open-market operations to outright financial repression. About rational bubbles, see Tirole (1985) and Santos and Woodford (1997). Hellwig and Lorenzoni (2009) deals with rational bubbles in international debt: it shows that it can only be sustained is the economy is dynamically inefficient, so that the gain from defaulting is not too high.

[4]On the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965)

[5]This is because bubbles can in that case grow at a rate higher than the rate of interest, even with a constant fraction of optimists in the population: their wealth grows at rate g, while the interest rate is $r < g$. Dynamic inefficiency thus provides a strong rationale for buying overvalued assets and chasing capital gains rather than dividends.

in a more unpredictable way.[6] Still on the positive front, dynamically inefficient economies have many of the properties of Keynesian economies as discussed by Geanakoplos (2008): in particular, they are isomorphic to Arrow-Debreu economies where markets do not clear at infinity. They therefore leave room for "animal spirits" to determine prices and drive business cycles.

But the real world provides the empiricist with multiple interest rates r to choose from. Should he use the safe interest rates on government bonds, in which case he would conclude that the economy is strongly inefficient?[7] Or should he use returns on equities, which almost always exceed the rate of growth?[8] Abel et al. (1989) extend the overlapping-generations capital accumulation model to account for more general production functions, in particular stochastic, with risk-varying interest rates. They conclude that the following sufficient criterion should be looked at: if capital income always exceeds investment, then the economy is in a dynamically efficient state. If investment to the contrary always exceeds capital income, then the economy is dynamically inefficient.[9] Empirically, Abel et al. (1989) find that this sufficient criterion for dynamic efficiency is satisfied by a wide margin for the United States $(1929-1985)$ and 6 other advanced economies $(1960-1985)$.

In contrast, I find that sufficient conditions for dynamic efficiency are verified for none of the advanced economies. To the contrary, Japan and South Korea verify the criterion for dynamic inefficiency.[10] And so do most advanced economies, including the United States, across a reasonable range of parameter estimates - in particular if one assumes that average Tobin's q is in the range of 1.5, an underestimation according to most analyses into this issue. For Australia and Canada, dynamic inefficiency is confirmed if Tobin's q is just a bit higher than 1. To the least, even taking a very conservative value of 1 for Tobin's q, that is assuming that monopoly rents and decreasing returns are nowhere present, dynamic inefficiency cannot be rejected using Abel et al. (1989)'s criterion. Moreover, taking the dynamic inefficiency of Japan as given (see Figure 1, which shows that investment has always been higher than capital income), an arbitrage argument suggests that other economies with which Japan has opened its capital account are dynamically inefficient too. Wouldn't capital otherwise flow out of Japan even more than it does?

Dynamic inefficiency could help explain a number of macroeconomic and financial stylized facts. A microeconomic counterpart to macroeconomic dynamic inefficiency is the claim in Fama and French (2002) and Campbell (2003), that firms seem to sink resources, since they

[6] For example, when the young buy overvalued houses from the old, it is effectively a transfer of wealth from the young to the old. Note that the bailing out of the financial sector also amounts to such a transfer that the private sector had been achieving on its own until the housing bubble collapsed.

[7] Interest rates on 10-year US Treasury Inflation Protected Securities (TIPS) are now negative.

[8] However, calculating returns on equity from stock market returns is a contaminated test of dynamic efficiency because dynamic inefficiency creates the potential for rational bubbles, which increase the rate of return to capital through capital gains. In particular, Fama and French (2002) note that the income return on book equity is much lower than the average stock return. More on this below.

[9] This criterion is actually not so different from the $g > r$ criterion. In steady-state, investment required to maintain the capital-over-output ratio is gK, while income coming from capital is $r_a K$ on average, with r_a being the average r_a obtained from investment. Hence, condition $gK > r_a K$ in all periods implies $g > r_a$. What Abel et al. (1989) therefore teach us is that one should look at the *average* interest rate in the economy, rather that at the *safe* interest rate.

[10] While one could argue that South Korea still is in a capital accumulation phase, it is certainly not the case for Japan.

get lower returns on dividends than is implied by their own measured cost of capital[11] (note that dynamic inefficiency, strictly speaking, arises when returns from dividends are lower than the rate of growth, which is usually more restrictive than the previous condition). It can explain financial instability, as with dynamic inefficiency, asset prices are no longer pinned down.[12] Linked to the possibility of rational bubbles, equity prices can command an equity premium over finitely-lived assets, if an inception of rational bubbles has increased their rate of return by higher-than-r returns from capital gains. Finally, among other examples (see Section 3), dynamic inefficiency can help shed a new light on the Japanese "lost decade", during which investment was substantially scaled down but consumption did not go down so much.[13]

The main reason for the difference between my results and Abel et al. (1989)'s results is new data from a recent release of a harmonized system of national accounts by the OECD, in particular with a different treatment of mixed income.[14] I also use more detailed data concerning land rents, which differs quite substantially from the one used in Abel et al. (1989). By means of an example, Abel et al. (1989) estimate land rents in Japan to be roughly equal to 5% of GDP, while they were according to both OECD and Goldsmith (1985)'s estimates, rather in the 17% of GDP range.

Related literature. To the best of my knowledge, no paper has reassessed dynamic efficiency since Abel et al. (1989) so far. The breadth of the literature which this paper speaks to is potentially very large, given the importance of dynamic efficiency for intergenerational transfers. From a methodological standpoint, there has been a renewed interest in national accounts data recently, in particular since the release of harmonized national accounts by the OECD. This has led to reassessing many common wisdoms. Gollin (2002) shows that labor income shares are not so variable across countries than economists once thought, and this paper tries to share the care he gives to the treatment of capital income. Caselli and Feyrer (2007) revisit Lucas' puzzle, showing that marginal product of capital is equalized across countries, once one accounts in particular for the effect of land and other non-reproducible resources. Piketty and Zucman (2012) use new balance sheet data to investigate the long run evolutions of wealth-income ratios over the courses of the nineteenth and twentieth centuries. A big advantage from using flows instead of stocks to study capital overaccumulation is that I do not have to make assumptions about the relative price of capital and consumption goods.[15]

[11] Fama and French (2002): "Most important, the average stock return for 1951 to 2000 is much greater than the average income return on book equity. Taken at face value, this says that investment during the period is on average unprofitable: its expected return is less than the cost of capital. In contrast, the lower estimates of the expected stock return from the dividend and earnings growth models are less than the income return on investment, so the message is that investment is on average profitable." ; and Campbell (2003): "if one uses average returns as an estimate of the true cost of capital, one is forced to the implausible conclusion that corporations destroyed stockholder value by retaining and reinvesting earnings rather than paying them out."

[12] The recursive equation $p_t = p_{t+1}/(1+r) + d_t$ admits an infinity of solutions depending on expectations at infinity. With dynamic efficiency, uniqueness is guaranteed through a transversality condition. Dynamic efficiency therefore comes naturally from optimization of an infinitely-lived agent, and dynamic inefficiency arises in overlapping-generations economies.

[13] Note that in an overlapping-generations model, GDP is no good measure for welfare, as more investment can lead to lower consumption always, and yet higher GDP.

[14] National Income and Product Accounts (NIPA) used by Abel et al. (1989) do not account for income of unincorporated enterprises, which include labor income.

[15] Similarly, it is tempting to back out an average return on capital from observed wealth income ratios as

From a substantive standpoint, this paper provides some support for Bernanke (2005)'s view that there indeed is a "savings glut" at the world level. It relates to a broad literature on the lack of safe assets, for example Caballero and Krishnamurthy (2006), Caballero et al. (2008) or Krishnamurthy and Vissing-Jorgensen (2012). The claim of this paper that the world does not so much lack safe assets, but lacks assets per say.[16]

The rest of the paper proceeds as follows. In section 1, I review very briefly Abel et al. (1989)'s sufficient conditions for dynamic efficiency. In Section 2.1, I reassess dynamic efficiency for the United States, using the same primary dataset as Abel et al. (1989) did use - that is, the National Income and Product Accounts - in order to highlight what differs in my calculations and leads me to a different conclusion. I then review dynamic efficiency in other advanced economies (15 additional countries) in Section 2.2. These two sections will lead to the conclusion that Japan and South Korea are dynamically inefficient, while dynamic inefficiency cannot be rejected for any advanced economy. Section 3 tries to go further at the cost of more assumptions; I let the reader make his mind about dynamic inefficiency of the other 12 advanced economies. Finally, I review in section 3 some stylized facts consistent with dynamic inefficiency, and I discuss some potential caveats to this study.

1 Sufficient conditions for dynamic efficiency

I here briefly review Abel et al. (1989)'s result that one should compare aggregate capital income and investment to assess dynamic efficiency. The setup generalizes Diamond (1965) to an economy with uncertainty, and a very general production technology.

There are overlapping generations of agents living for 2 periods: they are young then old. Time is discrete $t = 0, 1, 2, ...$. Agents have a Von-Neumann Morgentern utility function. There are L_t agents in each cohort. Young supply 1 unit of labor inelastically, get wage w_t, buy shares s_t of market portfolio V_t, and sell shares to young when old. Utility is additively separable across time for simplicity. Individual born in t solves :

$$\begin{aligned}
&\max_{s_t}\{u(c_t^y) + \mathbb{E}_t v(c_{t+1}^o)\} \\
&\quad \text{s.t.} \quad c_t^y = w_t - V_t s_t \\
&\quad \text{s.t.} \quad c_{t+1}^o = (D_{t+1} + V_{t+1}) s_t
\end{aligned}$$

measured for example by Piketty and Zucman (2012), and capital income: it would be Capital Income as a % of GDP divided by Wealth Income ratios as a % of GDP. This approach is however misleading, as not all future capital income is capitalized in today's capital values, like revenues from future ideas and companies. For example, assume that assets are trees decaying at a rate δ, giving a first dividend equal to R, and that new ideas come up each period to replace those decaying investments, as in Tirole (1985). Then the method outlined above would lead one to overstate return on capital by an amount δ, since the capitalized value of assets would be $R/(r + \delta)$ and capital income would be R in each period.

[16]However, the two theories have a lot in common. With dynamic inefficiency, there is a shortage of assets and so rational bubbles can form naturally as discussed above. These bubbles increase the supply of assets but as they are subject to a coordination problem, this supply is not "safe". Agents therefore would ideally need assets that they perceive as safe, even though they are valued above their fundamental values. In Krishnamurthy and Vissing-Jorgensen (2012), US Treasuries play this role.

The standard first-order condition for this maximization problem is:

$$\mathbb{E}_t \left[\frac{v'(c^o_{t+1})}{u'(c^y_t)} \frac{V_{t+1} + D_{t+1}}{V_t} \right] = 1.$$

Total consumption at date t is given by:

$$C_t = L_t c^y_t + L_{t-1} c^o_t.$$

Asset market clearing implies that $L_t s_t = 1$.

Defining profit: $\pi_t = Y_t - \frac{\partial F}{\partial L_t} L_t$, and investment I_t as being consumption's complement in output, Abel et al. (1989) prove the following proposition:

Proposition 1 (Abel et al. (1989))**.** *A sufficient condition for dynamic inefficiency is* $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - I_t \leq -\epsilon V_t$. *A sufficient condition for dynamic efficiency is:* $\exists \epsilon > 0, \forall t, \pi_t - I_t \geq \epsilon V_t$.

The intuition is pretty straightforward: the economy is dynamically inefficient if it is always investing more than it is getting from capital income, or sinking resources each period (at the steady state growth path). To the contrary, it is efficient if it is always investing less than it is getting out. The next section looks at this criterion again for the US economy first, and at other economies next; because the data sources are not the same for the two (the US has its own national accounting system, the NIPA, which Abel et al. (1989) use).

2 Reassessing dynamic efficiency

2.1 In the United States

In this section, I investigate dynamic efficiency in the United States assuming that Tobin's q is equal to 1. Since there are monopoly rents and decreasing returns to scale in the real world, this leads me to be too sanguine about dynamic efficiency (see Section 3.1 for different assumptions about Tobin q); yet the conclusion here will be that even with $q = 1$, dynamic inefficiency cannot be rejected in the United States. In order to pinpoint how my assessment differs from the seminal Abel et al. (1989), I follow them in using the National Income of Product Accounts , even though these are not the harmonized national accounts from OECD I use later.

I use the NIPA data maintained by the Bureau of Economic Analysis (BEA) to compare Gross Private Domestic Investment on the one hand, and Gross Capital Income on the other hand. While investment is available as a series, Gross Capital Income has to be calculated. Again, I follow their methodology in adding profit (including taxes on profit), rental income, interest income, capital income of proprietors, and private Capital Consumption Allowances (which are the difference of total and government Capital Consumption Allowances). On Figure 2 I compare data obtained from their Tables with data I calculate from today's series of the NIPA. I do not systematically over or understate capital income in any way, and the fit is good.

As these results are often remembered, capital income is about 25% of GDP, while investment is 15% of GDP. Therefore, dynamic efficiency seems to be satisfied by a wide margin for the United States. Yet there are 2 ways in which these calculations are being too sanguine about dynamic efficiency of the US economy. The first is that entrepreneurial income is not properly accounted for, as unincorporated enterprises are not taken into account in the NIPA, unlike in OECD mixed income. Second, because land rents are a bit higher than 5% of GDP as revealed by land values. I make both adjustments in turn:

- **Entrepreneurial income.** Quoting Mead et al. (2004), "Some aggregates exist in one system but not in the other. For example, NIPA corporate profits and personal income do not have precise counterparts in the SNA, and the SNA concept of "mixed income" - that is, the residual business income of unincorporated corporations that is attributable to labor and to capital has not been implemented in the NIPAs, pending a review of the sectoring of unincorporated businesses." NIPA only accounts for the income of proprietors, while OECD notion of mixed income includes also that of unincorporated enterprises. In other words, NIPA misses part of mixed income. How should I attribute mixed income between capital and labor income? This is extensively discussed in Gollin (2002), who goes as far as attributing the entirety of mixed income to labor. For robustness, I do not take such an extreme stand here: I only impute 2/3 of mixed income to labor. One should however bear in mind that this estimate is very conservative in many ways. First, this imputation is traditionally used since the work of Christensen (1971). At the time, proprietors' income was mostly that of farmers working in agriculture, where wages were relatively low; today, unincorporated enterprises use more skilled labor. in particular, doctors, lawyers operate in such structures. Furthermore, in most countries there exists a tax incentive to create an incorporated enterprise if the business is capital intensive; this is because unincorporated enterprise (e.g. LLC in the US) tax capital as personal income.[17] And finally, the share of capital in aggregate output is closer to 3/4 than 2/3. Figure 5 should really be seen as a higher bound on capital income excluding mixed income coming from labor.

- **Land rents, non-reproducible assets.** Land is a fixed non-reproducible factor, its return should therefore not be included to assess whether investment is on average productive or not.[18] Abel et al. (1989) use data from Rhee (1991).[19] They settle for 5% of GDP for the US, because it is consistent with an estimate of the aggregate value of land at 2/3 of GDP and a return of 8%.[20] Data on land rents is not directly available, as it is tied to residential structures, and both often trade as a bundle. I use two sources for

[17] More generally, there is an incentive to treat labor income as capital income for tax reasons. Capital income is thus always more likely to be overestimated than the contrary.

[18] In agricultural societies, one could argue that land needed to be somewhat maintained in order to remain productive. Today, land rents overwhelmingly consist of urban land, which has value because of economic geography considerations. For example, in the Alonso-Muth-Mills monocentric city model, higher land rents in the core are the exact counterpart of lower transportation cost.

[19] Rhee (1991) is using Denison (1962), itself updating Goldsmith(1959).

[20] Notwithstanding the very low assumed aggregate value of land in total assets, calculating land rents using returns is precisely what assessing dynamic efficiency is about, which is a bit circular. Instead, I will impute factor returns using their proportion in total assets.

calculating land rents in the United States: Goldsmith (1985) estimates a replacement cost of structure and attributes the remainder to land., which leads him to impute 25% of total assets to land (this is consistent with Davis and Heathcote (2007)).[21] Imputing land rents in this way leads to the results in Figure 6. Note that land is not the only non-reproducible asset: natural resources are another one. For the sake of conservativeness, I will however not make an attempt at substracting them from capital income.

2.2 In other countries

As in section 2.1, I hereby assume that Tobin's q is equal to 1, which goes against the conclusions of this paper. I compare capital income and investment for 15 additional countries (those for which OECD releases Gross Investment and Capital Income series): Australia, Belgium, Canada, Denmark, France, Germany, Hungary, Italy, Japan, Norway, Russia, South Korea, Sweden, Switzerland, United Kingdom.[22]

Reproducing. Abel et al. (1989) investigate dynamic efficiency in England, France, Germany, Italy, Canada, Japan. Their results are in Table 3 to the paper, which are plotted on Figure 9 as the difference between gross profit and gross investment. But they do not account for mixed income and land, which together account for about 9% of GDP. Note that with the Abel et al. (1989) data, Japan and Germany could already not be confirmed as being dynamically efficient, since investment is not lower than capital income over the whole period.

Updating. Land rents and mixed income do vary a lot across countries. OECD provides estimates of both for many countries. In contrast, Abel et al. (1989) used 5% of GDP for every country for lack of better data. When not available, I complement land price data with Goldsmith (1985) estimates from Comparative National Balance Sheets. Since Goldsmith (1985) does not provide data on land since 1978, I assume that land shares were constant ever since, and take the lowest of the 1973 or 1978 to be more conservative. These details are discussed more precisely in Appendix 3. Moreover, I perform a number of robustness checks in Section 3.3.

Results. I present in the main text the most inefficient economies according to OECD data: Japan and South Korea. I will not put too much of an emphasis on South Korea as it has developed only recently. The criterion for dynamic efficiency applies only at the (stochastic) steady-state, after the period of capital accumulation. Since I do not always have much data on mixed income, I present both capital income correcting for the value of mixed income and capital income containing mixed income. More precisely, capital income containing mixed income is an over-estimation of capital income coming from investment, but an under-estimation of capital income containing mixed income. This is because I calculate the value of land rents through the proportion of land in non-financial assets; so that I overestimate land rents when using capital income containing labor income from mixed income. More

[21] See Appendix 3 for a discussion on land data.

[22] I drop Mexico from the sample, because it has relative income to the US (in purchasing-power parity) significantly lower then the rest of my papers: about 30% on average. Dynamic efficiency is a steady-state concept, and such an emerging economy has not reached its steady-state of capital accumulation.

precisely, I use the following inequality:

$$\underbrace{\text{CapIncNolandNomixed}}_{\textbf{Capital Income}} = (\text{CapInc} - 2/3\text{mixed}) * \left(1 - \frac{\text{land}}{\text{assets}}\right) \leq \underbrace{\text{CapInc} * \left(1 - \frac{\text{land}}{\text{assets}}\right)}_{\textbf{Capital Income w/ mixed L}}$$

"Capital income with mixed Labor" is therefore a slight abuse of language in the graphs, only a fraction $1 - \text{land}/\text{assets}$ of mixed labor is actually included.

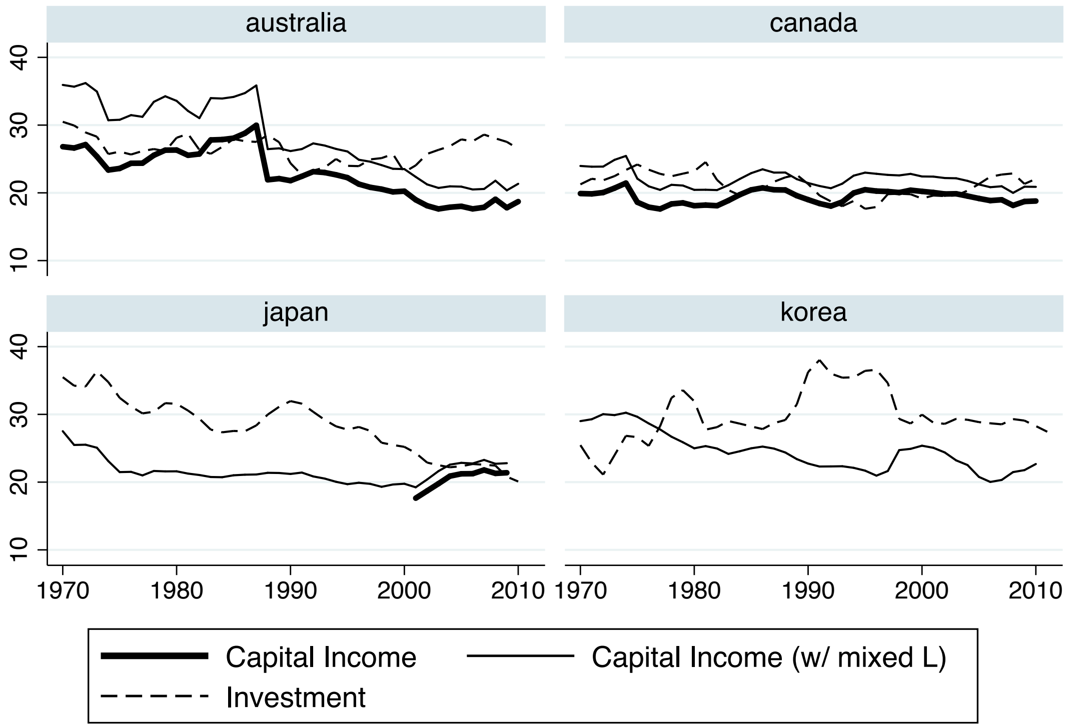
Description of the results. As can be seen on Figure 1, OECD does not provide data on mixed income for Japan before 2001. But the continuous thin line is always a higher bound on the thick line (capital income). Data from the OECD therefore suggests that Japan is inefficient. This contrasts starkly with Abel et al. (1989)'s results. The reason is that Japan has much higher land rents than the United States, because land is far more scarce. Therefore, by assuming that the United States have 5% of GDP in land revenues, Abel et al. (1989) strongly underestimate Japanese land rents. Data for mixed income in South Korea unfortunately is not available, but capital income including mixed income has been lower than investment since 1980, suggesting strong inefficiency. However, as discussed earlier, South Korea might already be in a stage of capital accumulation. Finally, Australia and Canada have low capital income compared to investment (excluding a few years in the eighties for Australia, and the nineties for Canada). This is surprising as Australia and Canada are not usual suspects for capital over-accumulation. Finally, Figures 13, 14 and 15 show that dynamic inefficiency cannot be rejected for any country (France, Germany, Hungary, Italy, Russia, Norway, Sweden, Switzerland, Beigium, United Kingdom).

3 Further calculations

From the upper left hand graph of Figure 1, one could boldly conclude that every advanced economy having an open financial account with Japan is dynamically inefficient. The reasoning is the following. In the absence of financial frictions, any dynamically efficient country could borrow from Japan and invest at a higher rate than Japan.[23] Therefore, a simple arbitrage argument would have all countries be inefficient if one of them is. But why is it then that investment is not always higher than capital income in other advanced economies as well? This section is about discussing other factors influencing the calculation of capital income, without necessarily being able to take a quantitative stance on these factors. In subsection 3.1, I argue that Tobin's average q is significantly higher than 1, which has led me to be too sanguine about dynamic efficiency. In subsection 3.2, I review other reasons which might have led me to be too sanguine about dynamic efficiency. In subsection 3.3 I do the opposite and examine the robustness of my calculations to other assumptions, notably about government investment.

[23] In practice, Japan has a current account surplus but it still is inefficient, perhaps less so today.

Figure 1: ASSESSING DYNAMIC EFFICIENCY IN AUSTRALIA, CANADA, JAPAN, SOUTH KOREA



Source: OECD, Kuznets (1985), and author's calculations

Notes: The dotted line represents Gross Capital Formation as a % of GDP. The thick continuous line is Capital Income as a % of GDP (including Capital Consumption Allowances), excluding land rents and mixed income coming from labor. Those two are to be compared to assess dynamic efficiency. Because data on mixed income is not always available, I also plot as a thin continuous line Capital Income excluding land rents but including labor income in mixed income (thus overstating "economic" Capital Income). "Korea" is South Korea. Data comes from OECD.

3.1 Taking into account Tobin's q

Extension of the Abel et al. (1989) model to decreasing returns

This model differs with Abel et al. (1989) in the production function $Y_t = F(I_{t-n}^{t-1}, L_t, \theta_t),$, which has decreasing returns. ($I_{t-n}^{t-1} = (I_{t-1}, ..., I_{t-n})$) Defining profit: $\pi_t = Y_t - \frac{\partial F}{\partial L_t} L_t$, and pure profit:

$$\pi_t^p = Y_t - \frac{\partial F}{\partial L_t} L_t - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

Proposition 2. ***(Decreasing returns.)*** *With decreasing returns to scale, a sufficient condition for dynamic inefficiency is* $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$. *Moreover,* $\exists \epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$ *is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is:* $\exists \epsilon > 0, \forall t, \pi_t - \pi_t^p - I_t \geq \epsilon V_t$.

Proof. See Appendix 1.1. □

Extension of the Abel et al. (1989) model to monopolistic competition

The setup is essentially the same as in the previous section, except that the consumption good is now a constant-elasticity of substitution aggregator of different varieties, and each firm produces one variety monopolistically. More precisely, both young and old consumption goods now are a CES of different varieties $\omega \in [0,1]$, such that:

$$c_t^y = \left[\int_0^1 c_t^y(\omega)^{\frac{\theta-1}{\theta}} d\omega\right]^{\frac{\theta}{\theta-1}}, \quad c_t^o = \left[\int_0^1 c_t^o(\omega)^{\frac{\theta-1}{\theta}} d\omega\right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

Dropping generation subscripts (everything is symmetric for old and young), the demand function for a generic good and the welfare-based price index are :

$$c_t(\omega) = \left(\frac{p_t(\omega)}{p_t}\right)^{-\theta} c_t.$$

$$p_t = \left[\int_0^1 p_t(\omega)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}$$

The model also differs from that of the previous section in that the environment is no longer competitive, but that of monopolistic competition. That is, every variety is produced by a monopolist with a constant-returns-to-scale[24] production function, defining as previously $I_{t-n}^{t-1}(\omega) = (I_{t-1}(\omega), ..., I_{t-n}(\omega))$:

$$y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t).$$

Note that θ_t is a productivity shock affecting all corporations equally. Denote the value function of minimizing the labor cost for an individual monopolistic firm ω:

$$\Lambda_t(y_t(\omega)) = \min_{L_t(\omega)} \left\{w_t L_t(\omega) \text{ s.t. } y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t)\right\}.$$

Again defining aggregate profit as:

$$\pi_t = \int_0^1 \left(\frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega))\right) d\omega,$$

and monopoly profit as:

$$\pi_t^m = \int_0^1 \left(\frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega))\right) d\omega - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i},$$

allows to state the following proposition:

Proposition 3. ***(Monopoly power.)*** *With monopolistic competition, a sufficient condition for dynamic inefficiency is $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$. Moreover, $\exists \epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$ is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is: $\exists \epsilon > 0, \forall t, \pi_t - \pi_t^m - I_t \geq \epsilon V_t$.*

[24]Here again, I could generalize to decreasing returns to scale, adding a "pure rent" component in addition to the "monopoly rents" profits, but this would only complicate the exposition.

Proof. See Appendix 1.2. □

Discussion of Tobin's q for the United States

Based on Figure 11 from Hall (2001) (updated by Philippon (2009)), one can notice that Tobin's q departs significantly from 1. Unfortunately, Tobin's q captures adjustment costs as well as potential monopoly rents and decreasing returns to scale.[25] Given Hall (2001)'s methodology, the starting value for Tobin's q is assumed to be 1. Moreover, given Hall (2001) adjustment cost model, which by construction minimizes the distance between Tobin's q and 1 (subject to constraints), average q is likely to be underestimated.

Note that in any case, Tobin's q cannot be lower than 1 for my purposes - that is, irreversibility of investment which potentially drives Tobin's q below 1 is of no interest. Decreasing returns to scale or monopoly power cannot be negative.

The discussion about Tobin's q, and why it can so persistently depart from 1, would lead us too far for the purpose of this paper. What is important is that while Hall (2001) work in the zero-rent framework, he repeatedly cites monopoly power as a potential microfundation for adjustment costs. Tobin's q might as well capture investment in intangibles that is not taken into account in official investment data - marketing costs are expensed for example, but they bring revenues in the future. In that case, it would be wrong to interpret the entirety of Tobin's q as consequential to the presence of rents. However, if investment was abnormally high in the 1990s and early 2000s as some have suggested to explain the "dotcom bubble", then most capital income would have materialized later, which would have dramatically increased the probability of dynamic inefficiency, at least in these years. Bond and Cummins (2000) question the importance of the intangible channel and instead point to irrational valuations from the part of investment. I believe there is more to the latter story than to the former, therefore I will never consider Tobin's q on the order of 3.[26]

Robustness to Tobin's q

Because of the difficulties outlined above, I do not want to take an affirmative stand on the value of Tobin's q. Given that Japan already provides us with the presumption that advanced economies may well be inefficient, I perform a sensitivity analysis using different parameters of Tobin's q.

The results for the United States are on Figure 16, for Japan, Anstralia and Canada on Figure 17. On Figure 12, I plot the fraction of years in which investment exceeds capital income. Graphically, dynamic inefficiency can be rejected for Tobin $q = 1$ in countries for which the line begins from the x axis. However, note that I here only use the thick line from previous graphs, which means I am missing many years for which those countries were actually inefficient, as suggested by the extrapolated thin line. Dynamic inefficiency cannot be rejected for Tobin $q = 1$ in countries for which the line begins from the y axis.

[25] Although adjustment costs can be microfounded by patent rights. (Hall (2001))

[26] Note however that the presence of rational bubbles is possible only with dynamic inefficiency. One could make the following reasoning: either rational bubbles exist and dynamic inefficiency is guaranteed, or they do not and Tobin's q should be taken at face value.

3.2 Other causes for overstatement of capital income

Capital income is likely to be overestimated for multiple other reasons not mentioned above because they are hard to quantify. However, some evidence suggests that some of them might be of first order significance. To get an idea of the orders of magnitudes involved, overstating capital income by 3% of GDP for the US leads to satisfy the sufficient condition for dynamic inefficiency assuming Tobin Q is equal to 1.

Incidence of corporate taxation. In the calculations above, taxes on profit are assumed to fall entirely on capital. Hence the revenue they raise is treated as capital income. If instead taxes on profit are borne by workers or consumers, as at least then taxes on profit are not capital income. Given the order of magnitude involved ($\simeq$ 2.3% of GDP), tax incidence is not a detail. In fact, if taxes on profit do not fall at all on capital income, then the sufficient condition for dynamic inefficiency is verified in the United States with Tobin q equal to 1.

Household production in financial services. Individual investors search for stocks, and put some effort in portfolio management. There is a cost to managing one's wealth, that is not recorded in the national accounts when it is not done professionally. Some individuals indeed spend a lot of time monitoring their financial intermediaries and finding more performing ones, doing their investment in stocks, etc. Other rent their real-estate assets, and provide the labor services of choosing tenants and collecting rents. All this implies that pure capital income tends to be over-estimated. As Piketty and Saez (2011) put it, these efforts should be viewed as informal financial services that are directly supplied and consumed by households; they estimate these financial services to be of the order of **2%-3% of GDP** at the very most.

Favorable tax treatment of capital income. As pointed out previously, it is very hard to measure the share of labor in mixed income. This is all the more true that capital income is less heavily taxed than labor income in most tax systems, so that entrepreneurs have a strong incentive to make their labor revenues appear as capital revenues. Hence, revenues from LBOs are usually treated as capital gains, or as revenue accruing to investment, while it usually employs a very qualified workforce to pick these investments and "beat the market".[27] In other words, management fees often understate the return to labor - part of financiers' wages are earned through capital gains. The public finance literature has only begun to investigate this issue (for example Piketty (2011) and Piketty and Saez (2011)).

Public debt and rational bubbles. Needless to say, advanced economies have very high levels of debt. To the extent that Ricardian equivalence does not hold in an overlapping generations model, private savings do not perfectly offset these public dissavings. Hence, dynamic inefficiency would likely even be more severe absent our extraordinary levels of public debt. Rational bubbles can similarly crowd out private savings and raise consumption. To the extent that they are a feature of the real world, rational bubbles lessen the severity of dynamic inefficiency.

Other rents. Only land rents have been taken out of capital income. But there are other physical rents, of which the World Bank maintains a data series (extensively used by Caselli

[27] Very often, changing the corporate finance structure of the company and loading it with debt helps benefit from the tax deductibility of debt interest.

and Feyrer (2007)). However, it might well be that countries have not reached a steady-state of their resource-extraction path (for example, US oil), so that excluding these rents would actually lead to an underestimation of capital income. For the sake of robustness, I do not attempt at such a calculation here, which would only strengthen my conclusions.

3.3 Robustness

Government investment. In Appendix 8, I take out government investment from Investment series, when available. I also take out Public Capital Consumption Allowances from Capital Income. Note however that such a calculation leads to be too sanguine about dynamic efficiency, as government investment mostly yields private benefits in the form of private capital income. Figures 21, 22, 23 and 24 give the results.

Data on land. One might worry that the "asset approach" yields to overstate the importance of land in value added, especially if land is subject to overvaluation due to animal spirits. This potential limitation applies only to 6 countries for which I use recent OECD data. I perform a number of robustness checks in this direction. In particular I assume that land shares stayed constant after 2000 - when the run-up in house prices began in most advanced economies.

4 Discussion

4.1 Consistent stylized facts

Feldstein and Horioka (1980) puzzle. A puzzle in international finance is that investment and savings are highly correlated in the cross section of countries. This puzzle has been named the Feldstein-Horioka puzzle: investment opportunities have no reason to be present where savings rate are also higher. Current accounts should make up for the differences, but they are relatively small compared to the differences in savings and investment rates across countries. All this theoretical analysis however assumes that capital is relatively scarce. With dynamic inefficiency, asset supply is in any case too low relative to asset demand, and so even an arbitrarily small amount of home bias translates into so high level of savings-investment correlation.

Crises. The severity of dynamic inefficiency seems to be strongly correlated with key macroeconomic events, like banking crises. For example, as the second and third quadrants of Figure 14 show, Norway and Sweden had a dynamic efficiency problem before their banking crises in $1991-1993$. Japan was the most severely inefficient of all countries in the 80s, and the "lost decade" corresponds to a period of declining investment and increasing consumption.[28]

4.2 Potential caveats

Externalities to capital accumulation. An important factor for the validity of this analysis that there be no externalities, or that those are negligible. In a capital accumulation

[28] Note that going from an above-Golden rule steady state growth path to a Golden-Rule one mechanically decreases output and leads to a recession, even though consumption increases.

model with externalities (e.g. Saint-Paul (1992)), capital income could understate returns to capital, and increasing public debt in case of dynamic inefficiency could lower consumption. Note however that capital externalities are usually a feature of human capital rather than physical capital.

Conclusion

"Search for yield", "scarcity of assets", "abundant liquidity", all these phrases point to the fact that there might well be too many savings chasing too few assets in the world. Because of high levels of public debt, older people are usually accused of not caring enough about their children. In contrast, this paper suggests that they still save too much, relative to the relatively few investment opportunities present in our economies.

Series from the Bureau of Economic Analysis start in 1929, just before the Great Depression; before that, national accounting was very rudimentary. But the difference between capital income and investment was similar in 1929 than it is today. Was capital also overaccumulated at the eve of the Great Depression, after the long period of capital accumulation in the nineteenth century documented in particular by Piketty (2011)?

In any case, dynamic inefficiency invites us to revisit many policy questions. In a world of too much capital accumulation, capital taxation, which is often thought of as a deterrent to capital accumulation, is perhaps not a bad idea after all.[29] At the same time, future capital taxes decrease the value of assets today, and so increase the problem of asset scarcity. Dynamic inefficiency also makes rational bubbles possible (Tirole (1985), Santos and Woodford (1997)): when assets are scarce, any real, financial or even monetary asset can become a locus for bubbles. Other social contracts such as pay-as-you-go systems to replace funded systems can also be Pareto-improving.

Solving the problem of dynamic efficiency is certainly not as straightforward as Diamond (1965) suggested. In practice, the market may have a hard time coordinating on so high implied levels of debt. This is all the more true that this debt is not an infinitely lived asset, and has to be refinanced from time to time. Moreover, it always is subject to the risk of default, even if Hellwig and Lorenzoni (2009) show that the costs of defaulting (being further excluded from the financial markets) far exceeds the benefits with dynamic inefficiency. Moreover, I suspect that markets can very well coordinate on other rational bubbles that are equally fragile; and that public debt does not have a special status in this respect. I leave this very important issue for future research.

[29] However note that Piketty and Saez (2011) recommend in their Appendix to use public debt for dealing with dynamic inefficiency, to restore the Golden Rule level of capital accumulation; and use capital taxes for redistribution motives. There is a dichotomy with 2 instruments and 2 objectives.

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1 Proofs

1.1 Proof of Proposition 1

I will go over the sufficient condition for dynamic inefficiency in detail, since the condition is less restrictive as in Abel et al. (1989). The proof for the sufficient condition for dynamic efficiency is very similar.

Proof. Assume that : $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$. Then by definition

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i} - I_t \leq -\epsilon V_t \leq -\epsilon I_t$$

Therefore:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment (so that it is resource-feasible), and the size of this increase be $\delta > 0$, $dc_0 = \delta$ small $\Rightarrow dC_0 = \delta L_0, dI_0 = -\delta L_0$. To make this change Pareto-improving, one has to make up for the decrease in output in the following periods by reducing investment as well. From the production function, production in period 1 decreases by $dY_1 = \frac{\partial F}{\partial I_0} dI_0$, and so for unchanged consumption $dC_1 = 0$, $dI_1 = \frac{\partial F}{\partial I_0} dI_0$. More generally, today's output will be reduced by the n previous reductions in investment of all vintages. The general formula is:

$$dI_t = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}$$

$$\Rightarrow \frac{dI_t}{I_t} = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}.$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since $\forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$, such a change is feasible for $\delta > 0$ sufficiently low. $\square$

Once can similarly extend Abel et al. (1989) to sufficient conditions for efficiency, redefining profit in an adequate manner.

1.2 Proof of Proposition 2

Again, I will only go over the proof for dynamic inefficiency.

Proof. Let us first calculate each firm's profit. Facing demand for its product as:

$$y_t(\omega) = \left(\frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t,$$

a monopolistic firm chooses $p_t(\omega)$ so as to maximize its profit, that is:

$$\max_{p_t(\omega)} \left\{ p_t(\omega) \left(\frac{p_t(\omega)}{p_t}\right)^{-\theta} y_t - \Lambda\left(\left(\frac{p_t(\omega)}{p_t}\right)^{-\theta} y_t\right) - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega) \right\}.$$

The firm charges a markup over marginal labor cost:

$$p_t(\omega) = \frac{\theta}{\theta-1}\Lambda_t'(Y_t(\omega)).$$

Monopoly profits for a single firm are therefore:

$$\pi_t^m(\omega) = \frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega)) - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega).$$

Total profits for a monopolistic firm, including returns to capital are:

$$\pi_t(\omega) = \frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega)).$$

Using that the returns on capital for each firm are equalized because it is supplied competitively, I can sum over ω to find:

$$\pi_t^m = \int_0^1 \pi_t^m(\omega)d\omega = \int_0^1 \left(\frac{\theta}{\theta-1}\Lambda_t'(y_t(\omega))y_t(\omega) - \Lambda_t(y_t(\omega))\right)d\omega - \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

Now as in the previous proof, assume that : $\exists\epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$. Therefore:

$$\exists\epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1-\epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment in each monopolistic firm (so that it is resource-feasible), and the size of this increase be $\delta > 0$, $dc_0 = \delta$ (per-capita consumption) small $\Rightarrow dC_0 = \delta L_0$, so that the aggregate decrease in investment must be $dI_0 = -\delta L_0$. Let us split this decrease in investment equally among firms so that $dI_0 = \int_0^1 dI_0(\omega)d\omega$. To make this change Pareto-improving, one has to make up for the decrease in output in each firm in the following periods by reducing investment as well. From the production function, production in period 1 decreases by $dY_1(\omega) = \frac{\partial F}{\partial I_0}dI_0(\omega)$ in each firm, and so for unchanged consumption $dC_1 = 0$, $dI_1(\omega) = \frac{\partial F}{\partial I_0}dI_0(\omega)$ in order to maintain the same level of production in each firm. More generally, today's output will be reduced by the n previous reductions in investment of all vintages. The general formula is:

$$\forall\omega \in [0,1], dI_t(\omega) = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}(\omega).$$

Summing, and because marginal returns to capital are equalized across firms:

$$dI_t = \int_0^1 dI_t(\omega)d\omega = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \int_0^1 dI_{t-i}(\omega)d\omega = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} dI_{t-i}$$

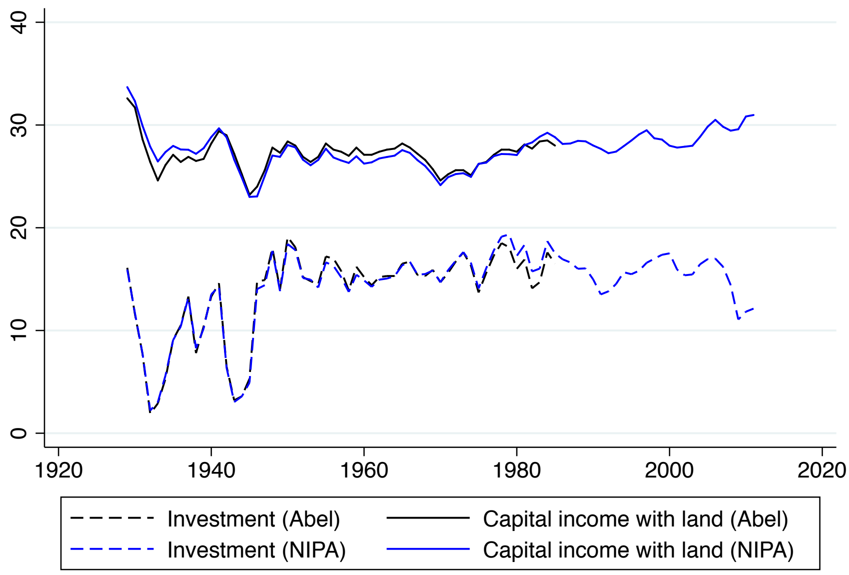
$$\Rightarrow \frac{dI_t}{I_t} = \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}.$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since $\forall t \in \mathbb{N}, \sum_{i=1}^{n} \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$, such a change is feasible for $\delta > 0$ sufficiently low.

□

2 Reproducing Abel et al. (1989) step by step for the United States

Figure 2: REPRODUCING AND UPDATING ABEL ET AL. (1989)



Source: National Income and Product Account (NIPA) and Abel(1989)'s calculations

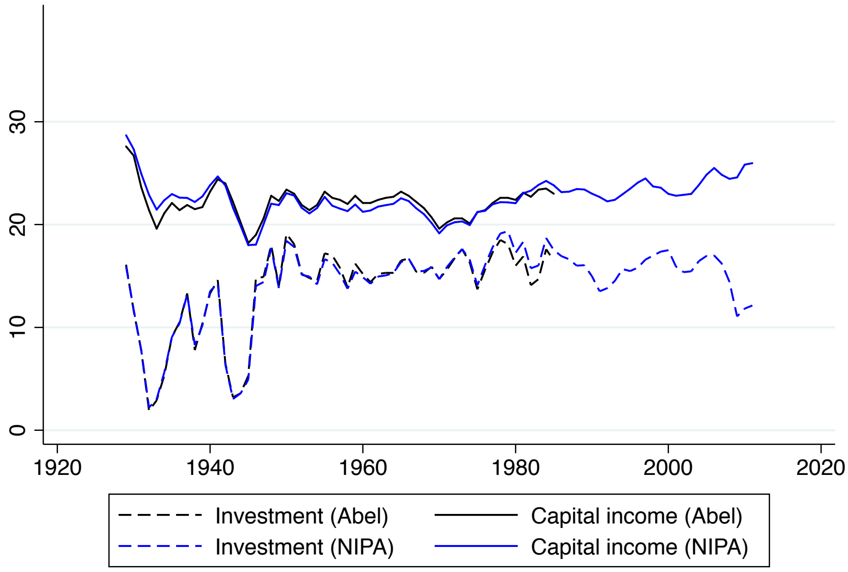
3 Data on land

Data on land comes mainly from two sources:

- OECD for 6 countries (Australia, Canada, Czech Republic, France, Japan, Korea). Data for OECD is plotted on Figure 7.
- Goldsmith (1985) *Comparative National Balance Sheets*, for 20 countries (with two dates 1973 and 1978).

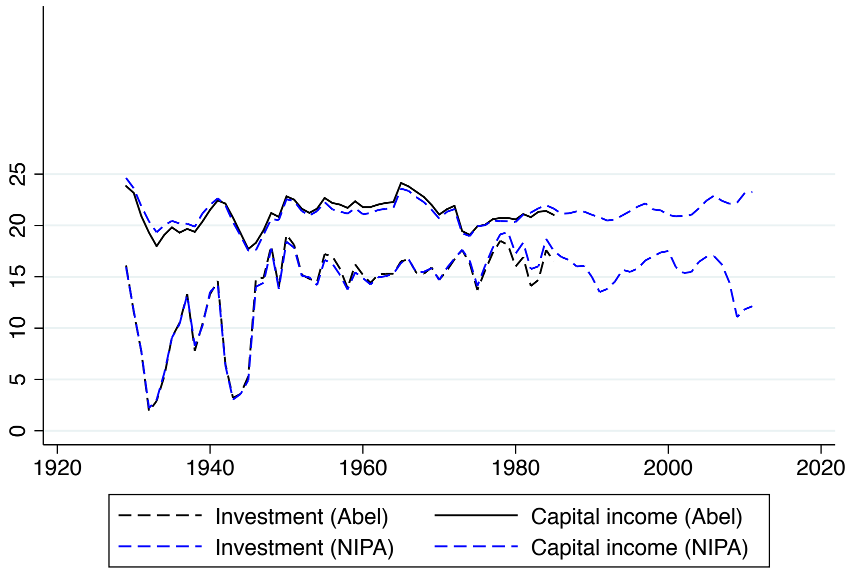
For other countries, I take the lowest value in 1973 or 1978 from Goldsmith for all years. This is very conservative, especially for the last decade. Furthermore, Goldsmith underestimates relative to OECD (although it displays a similar evolution).

Figure 3: Removing land rents at 5% of GDP



Source: National Income and Product Account (NIPA) and Abel(1989)'s calculations

Figure 4: Using OECD data for land rents



Source: NIPA, Goldsmith(1985),Abel(1989)'s and author's calculations

In order to impute land income, I use an asset approach. That is, I assume that assets produce revenues in proportion to their relative importance in the capital stock. As discussed in Kravis

Figure 5: Using OECD data for mixed income

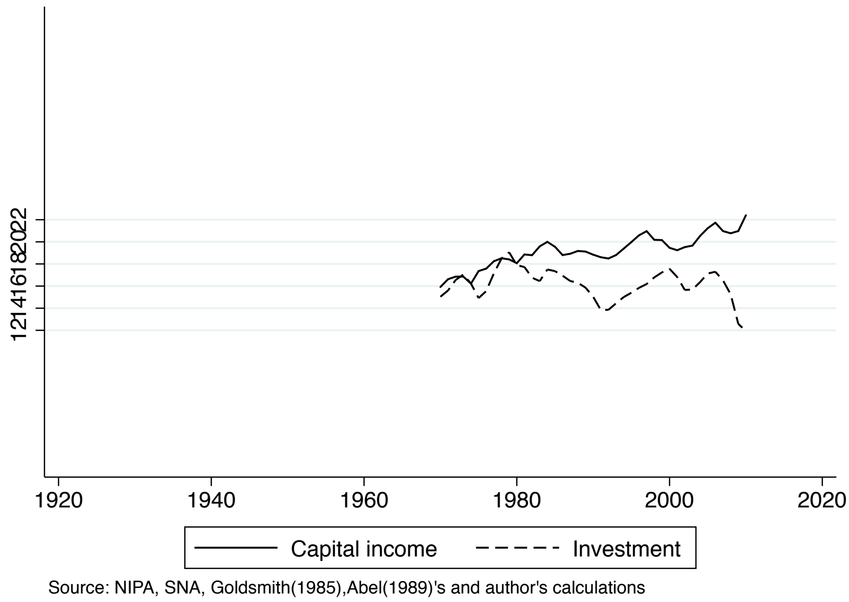
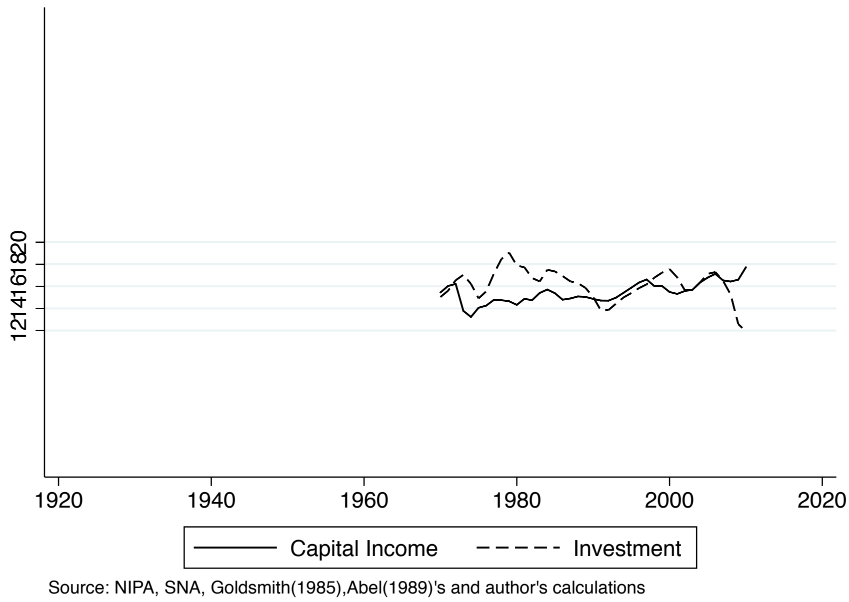


Figure 6: Preferred specification, assuming Tobin $q = 1$

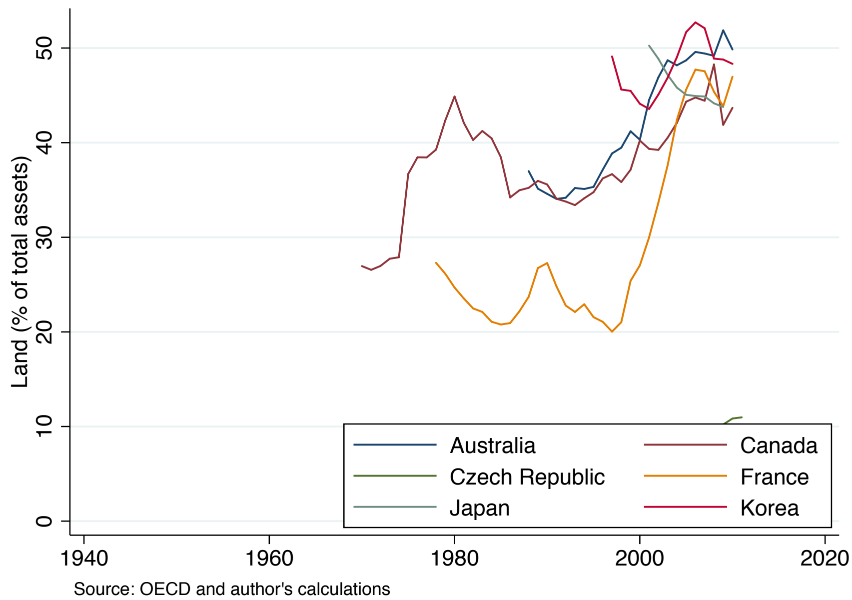


4 Reproducing Abel et al. (1989) step by step

4.1 United States

Abel et al. (1989) use data from the National Income and Products Accounts (NIPA). Raw data from Table 1 is reproduced in the upper left hand corner of Figure 8. Upper right hand corner updates this data with contemporaneous data given by the Bureau of Economic

Figure 7: Land as a % of total assets



Analysis. The numbers are very similar ; this confirms that I am using the same methodology for calculating capital income. In the bottom left hand corner, I use their estimate of land rents (5% of GDP), which I substract from capital income. Finally, I use more recent data on land rents in the bottom right hand corner.

4.2 UK, France, Germany, Canada, Italy, Japan

They then use gross investment and gross capital income coming from OECD database. Results from Table 3 are given in Figure 9. Note that even with their data, and taking an estimate of 5% of GDP for land rents and 4% of GDP for labor mixed income, dynamic efficiency could not be confirmed in Japan, Germany, United Kingdom, and Canada.

As shown on Figure 10, my estimates sometimes coincide, sometimes are higher for capital income than theirs. If anything, overstating capital income goes against the conclusions of this paper, and should only reinforce them.

5 Data on Tobin's q

6 Inefficiency as a function of Tobin q

This figure 12 summarizes the preceding graphs in $3D$. It plots the fraction of years for which investment exceeds capital income in the data, as a function of assumed Tobin q. When equal to 0%, the sufficient condition for efficiency is satisfied. When equal to 100%, the economy has unambiguously sinked resources into investment, and is dynamically inefficient.

Figure 8: MODIFYING ABEL AND AL. (1989), STEP BY STEP

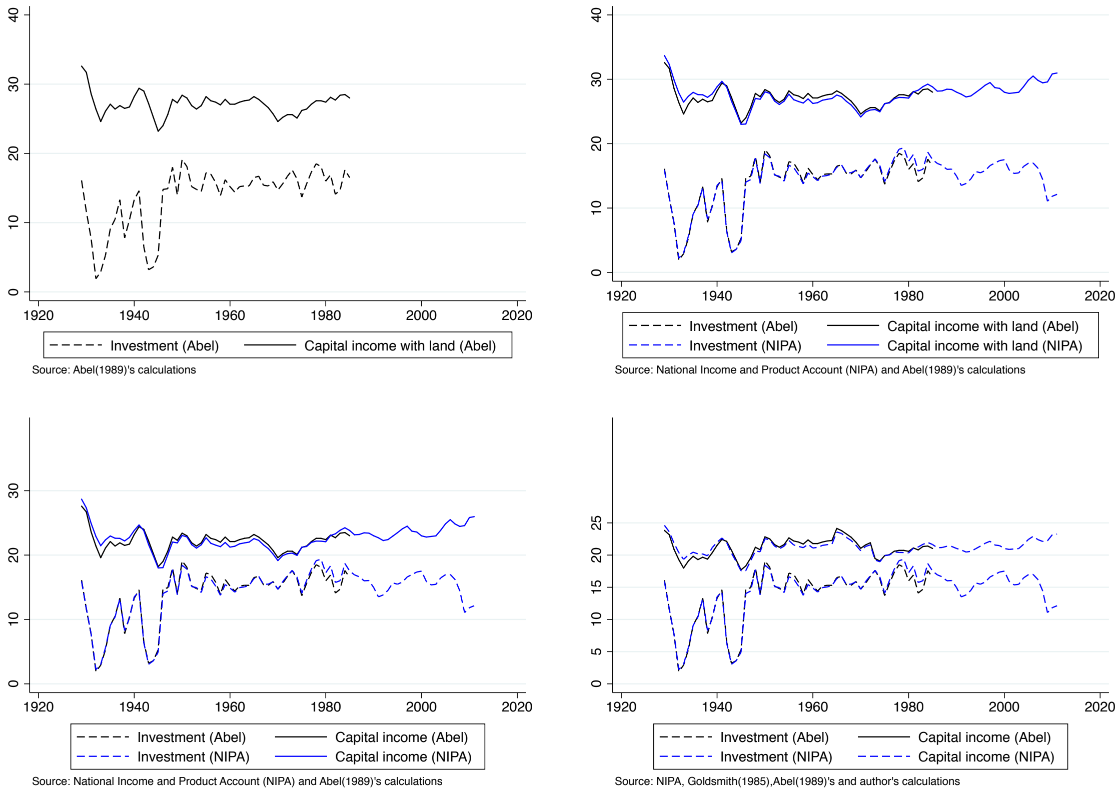


Figure 9: PLOTTING RESULTS FROM TABLE 3

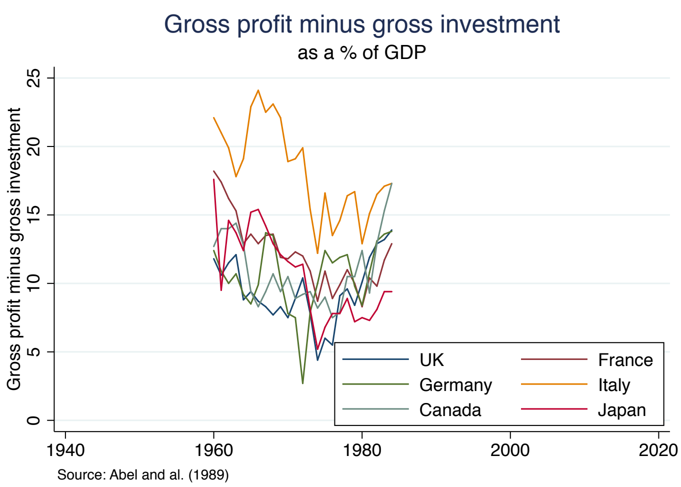


Figure 10: Reproducing for 6 OECD economies

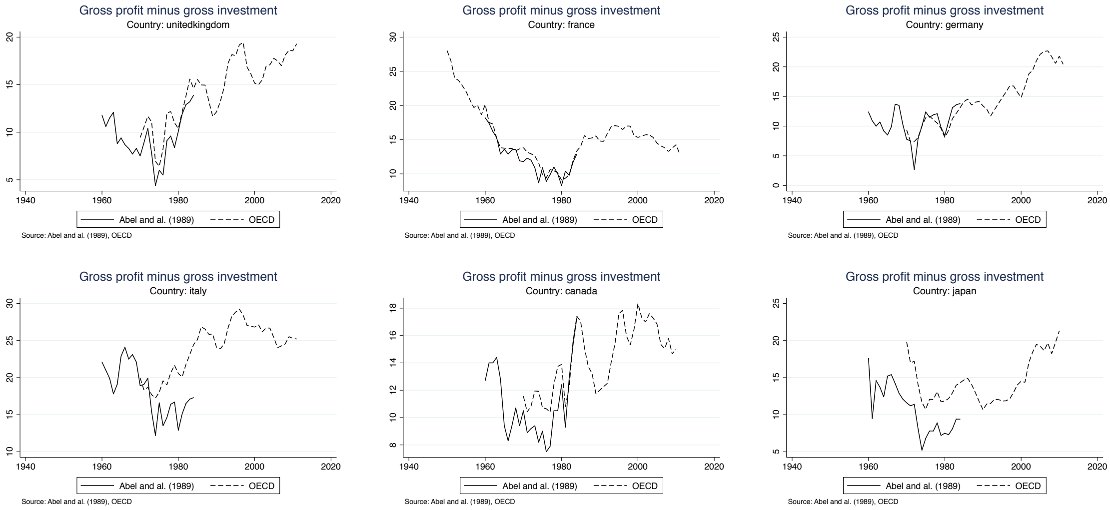
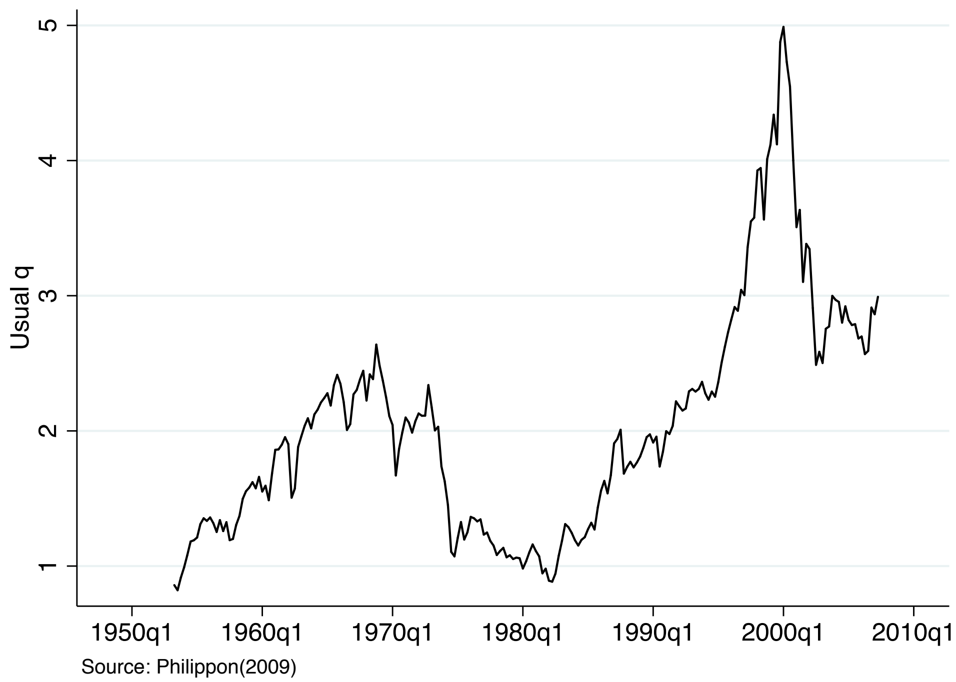


Figure 11: Tobin's q, using equity values.

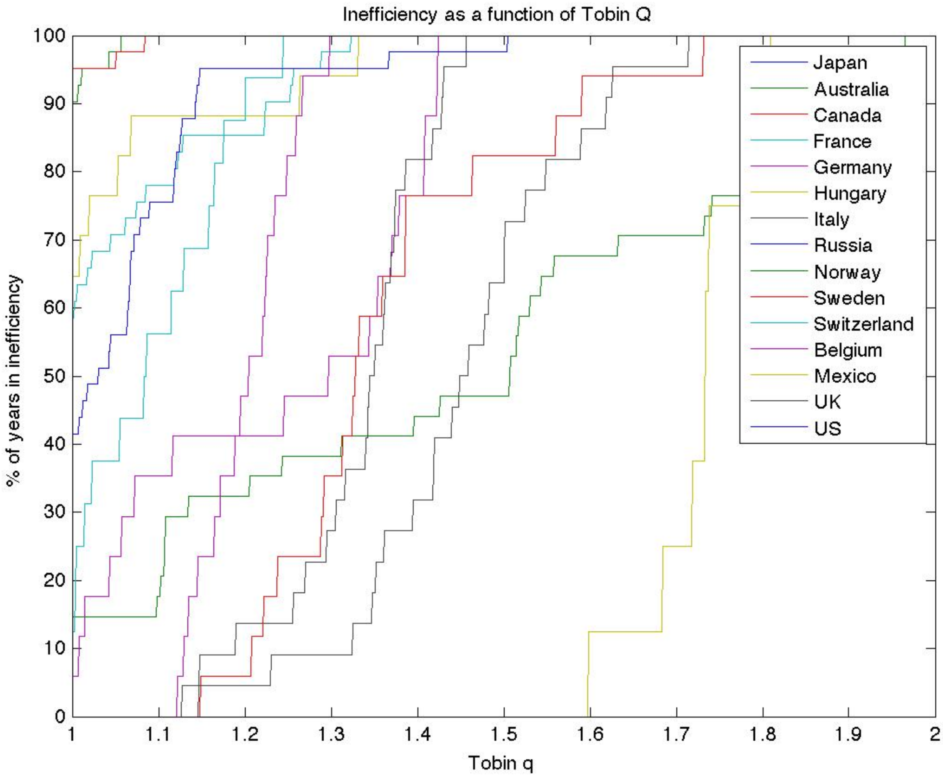


7 More countries

7.1 Tobin's q equal to 1

Capital income and investment in countries for which there is data are plotted in Figures 13, 14 and 15 (reminder: data for 4 inefficient economies is plotted in Figure 1).

Figure 12: FRACTION OF YEARS (IN %) IN WHICH INVESTMENT EXCEEDS CAPITAL INCOME



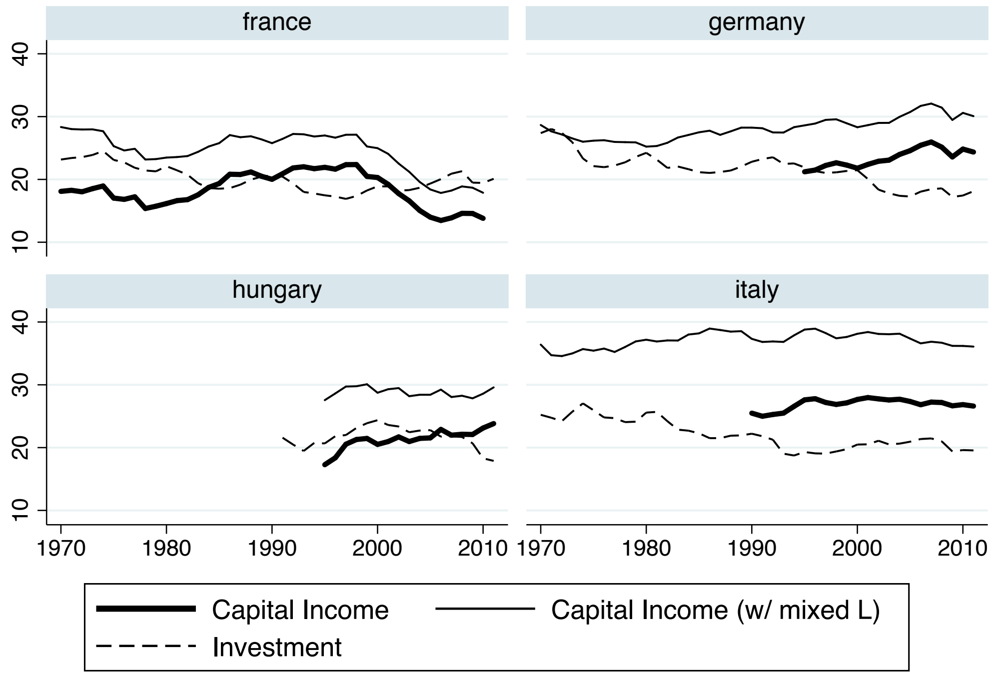
7.2 Varying the q of Tobin

Similarly, Tobin q is allowes to vary in Figures 18, 19 and 20. Reminder: inefficient economies are plotted in Figure 17.

8 Robustness: Government investment

As Abel et al. (1989), I have used private investment when using the Bureau of Economic Analysis (NIPA) data, and investment (including private and public) when using OECD data. This is because government investment series and capital consumption expenditures for government are not always available for all countries. However, I show here that government investment isn't driving dynamic inefficiency. In other words, it is not the case that government investment is so inefficient that it explains why capital income is low compared to investment. Moreover, there are many reasons to believe that private capital income includes some returns to government investment, as firms need roads to operate, for example. Results are show in Figures 21, 22, 23 and 24.

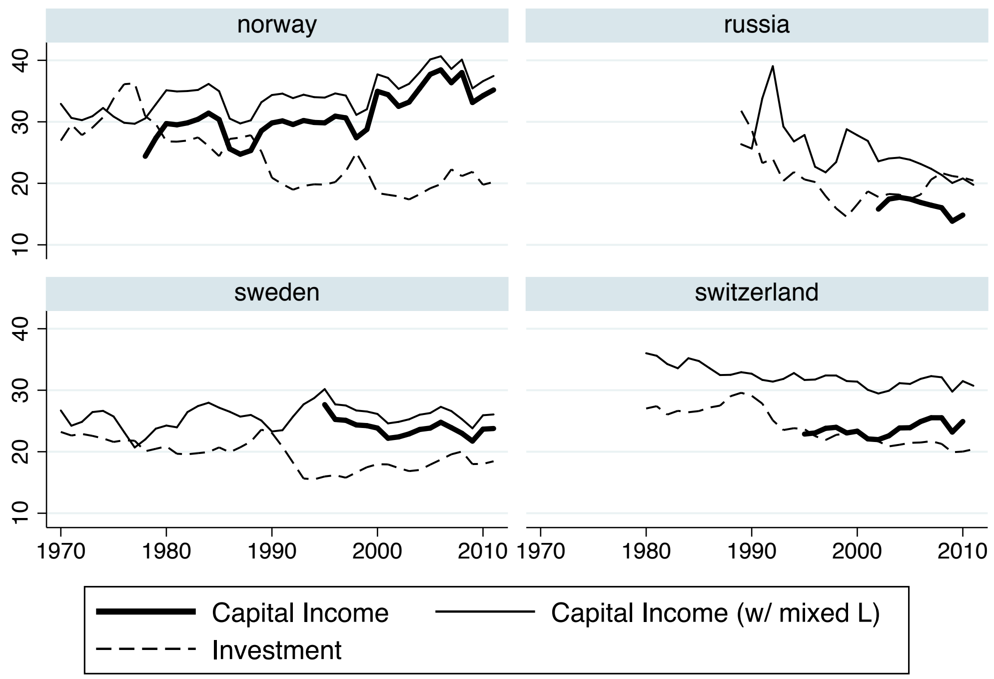
Figure 13: Assessing dynamic efficiency in France, Germany, Hungary and Italy



Source: OECD, Kuznets (1985), and author's calculations

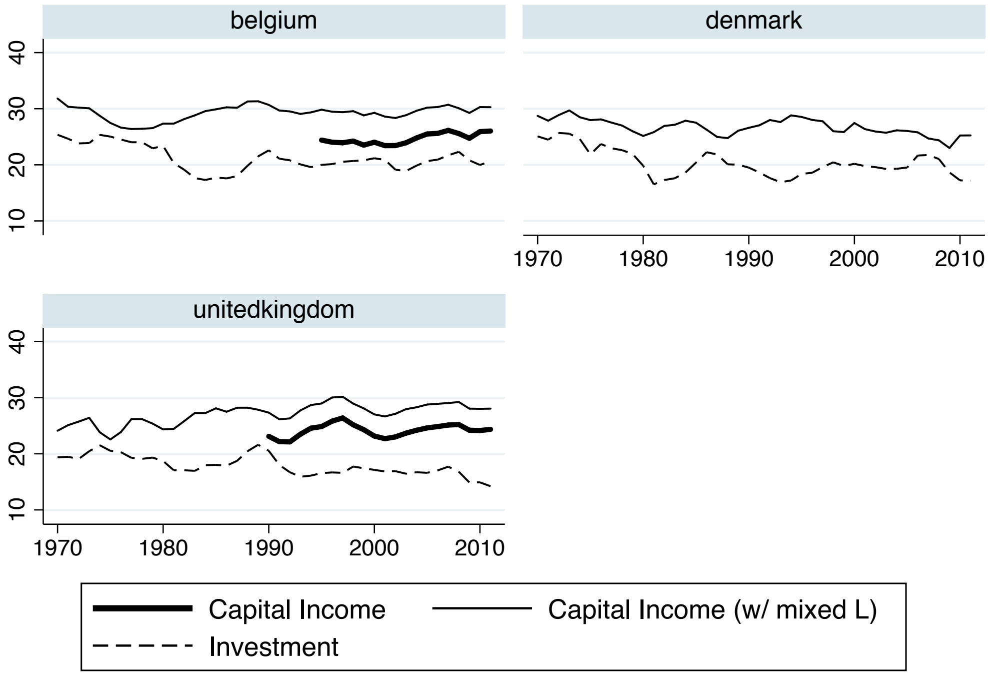
Notes: See Figure 1.

Figure 14: Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland



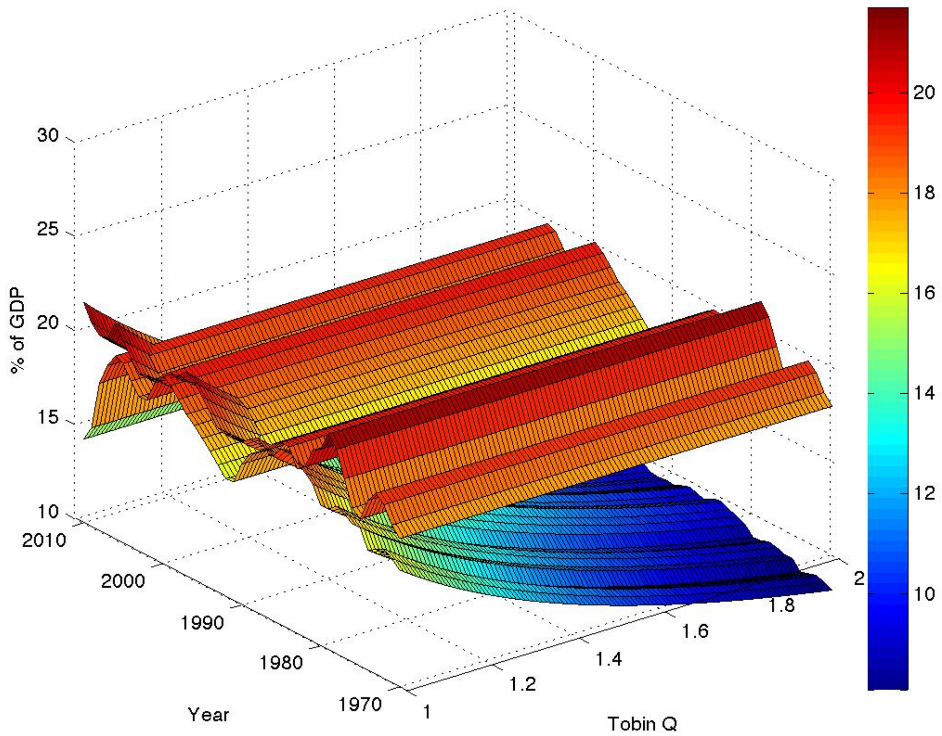
Source: OECD, Kuznets (1985), and author's calculations

Figure 15: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, DENMARK, UNITED KINGDOM



Source: OECD, Kuznets (1985), and author's calculations

Figure 16: Dynamic efficiency as a function of Tobin Q in the US



Notes: This 3D graph represents capital income and investment (as a % of GDP), capital income coming from investment being an hyperbolic function of Tobin's q. If average Tobin q is superior to 1.5, then capital income is lower than investment for all years, and therefore the US economy is dynamically inefficient. The data is from OECD.

Figure 17: Dynamic efficiency as a function of Tobin Q in Japan, Australia and Canada

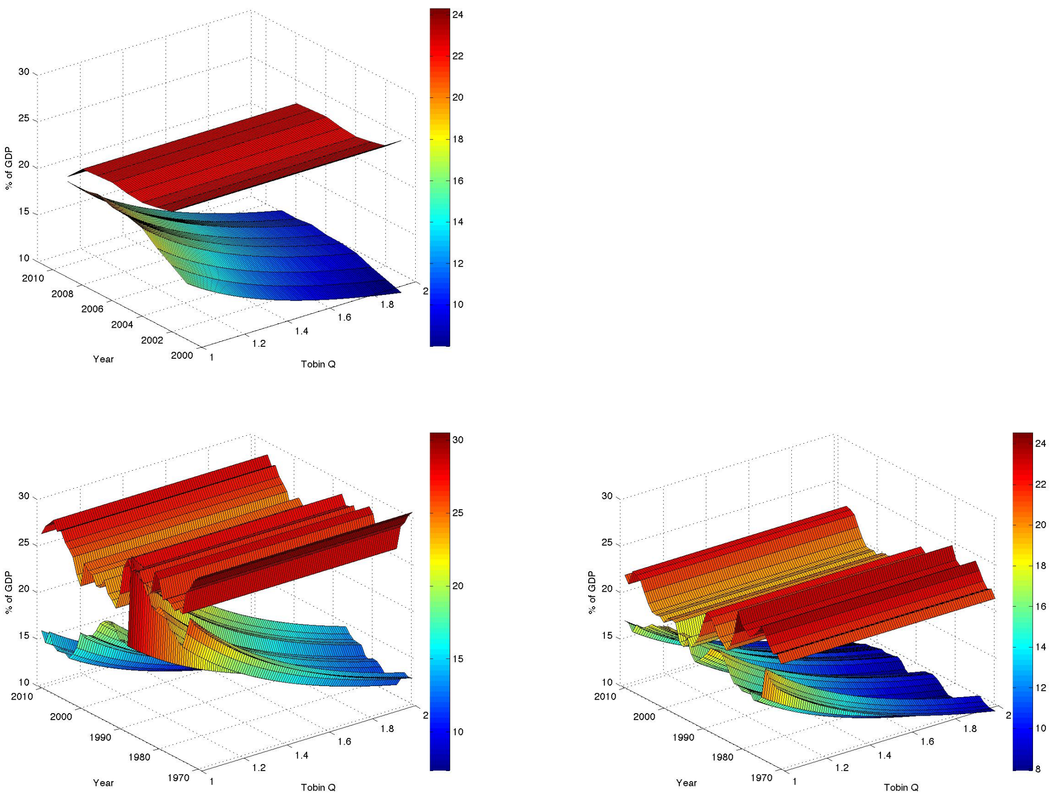


Figure 18: Assessing dynamic efficiency in France, Germany, Hungary and Italy

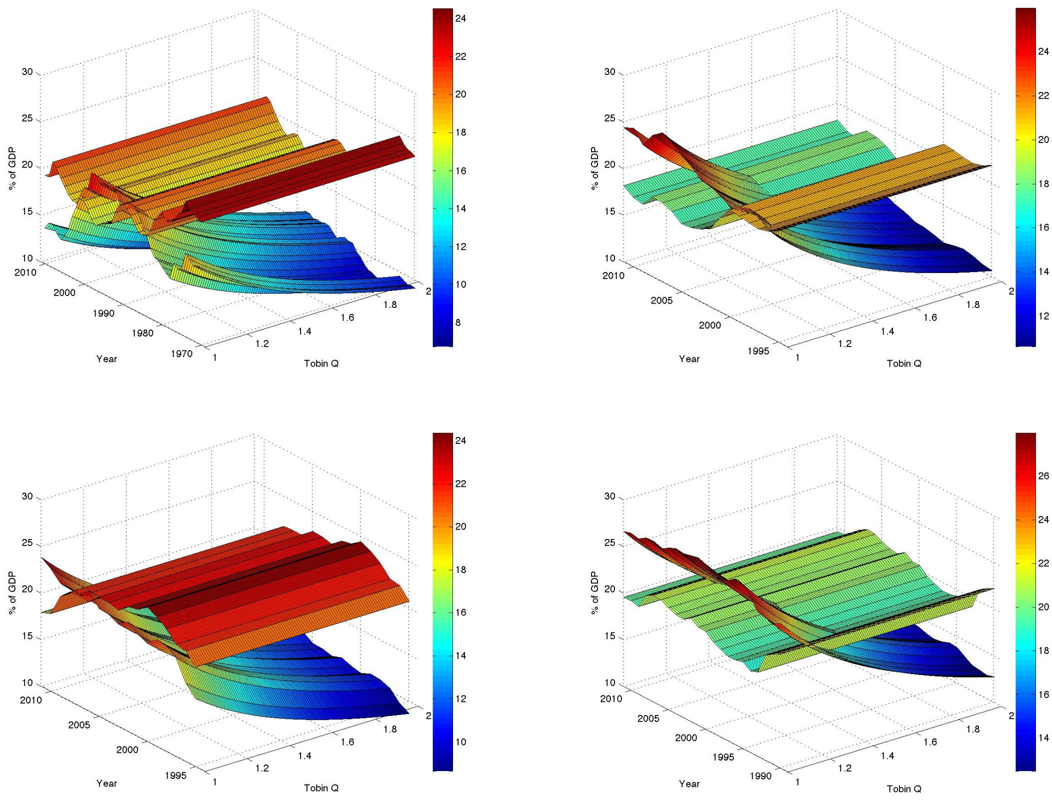


Figure 19: Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland

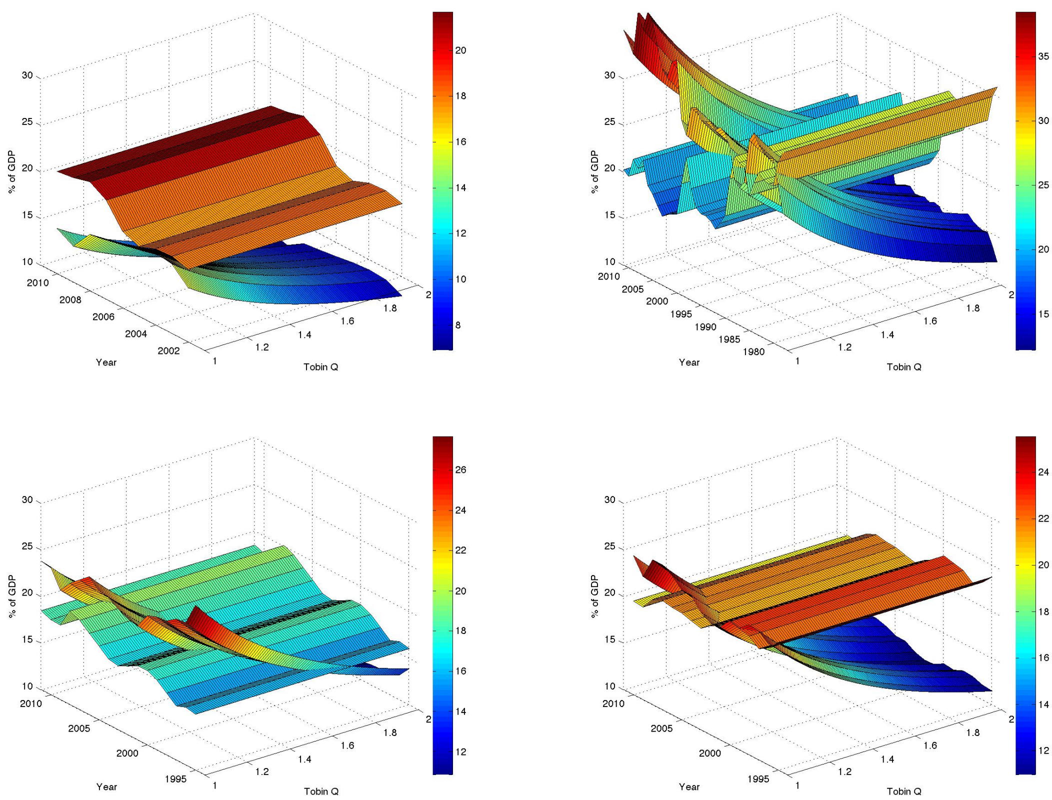


Figure 20: Assessing dynamic efficiency in Belgium, and United Kingdom

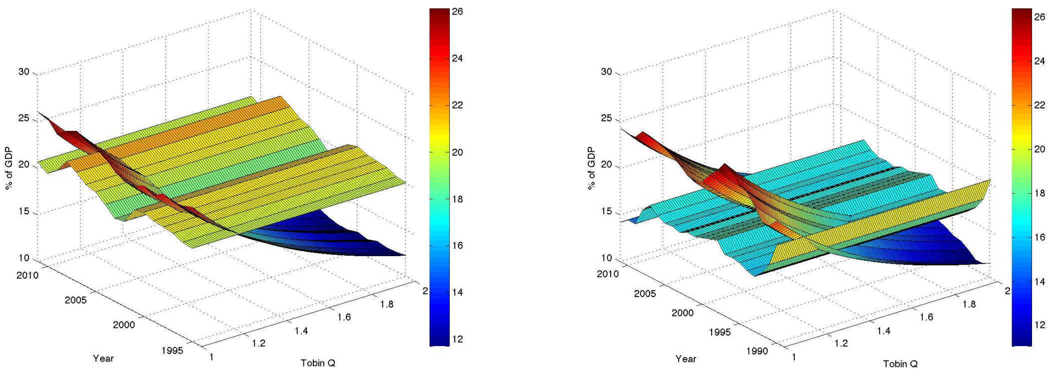
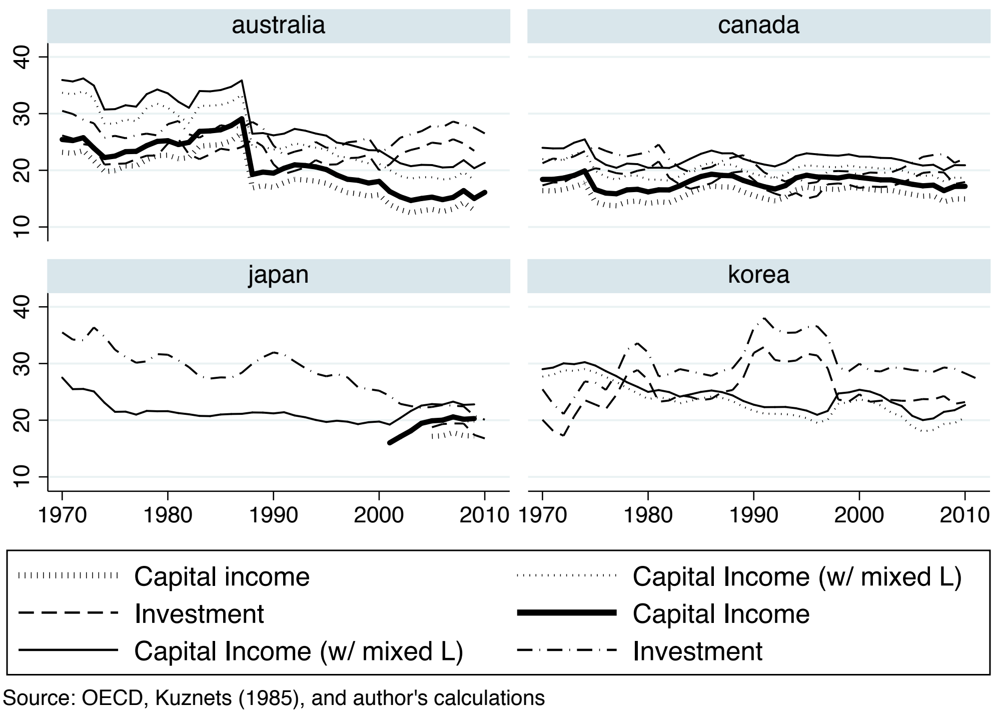
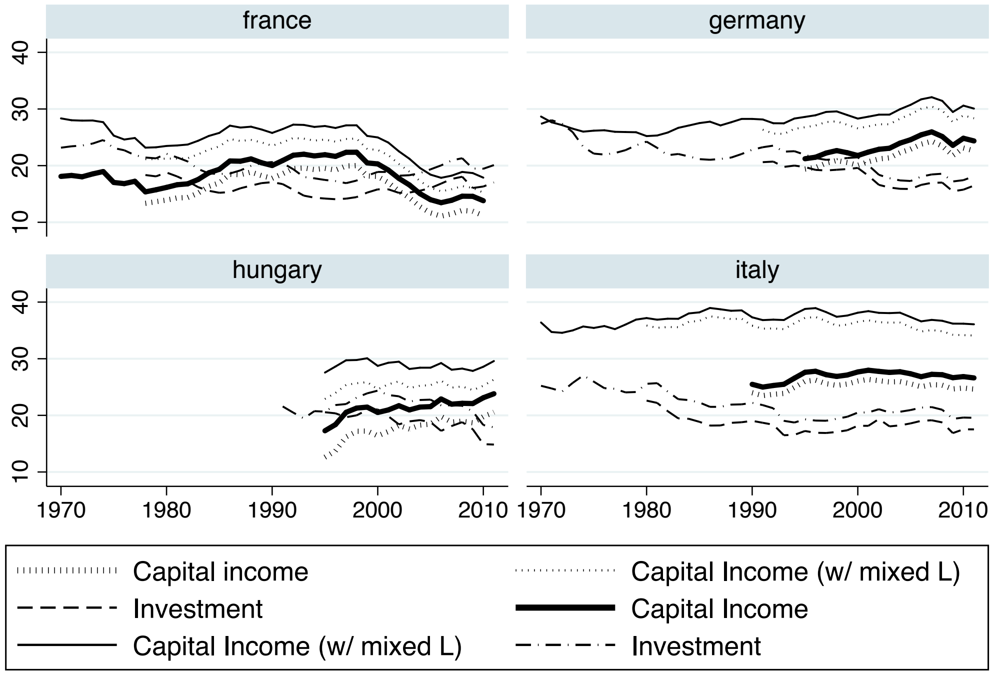


Figure 21: Assessing dynamic efficiency in Japan, South Korea, Australia, Canada



Notes: The dotted line represents Gross Capital Formation as a % of GDP. The dash-dotted line is total investment, excluding public investment. The thick continuous line is Capital Income as a % of GDP, excluding land rents and mixed income coming from labor. The thin continuous line is Capital Income excluding land rents but including labor income in mixed income "Korea" is South Korea. The thick dotted line is Capital income as a % of GDP excluding Government Consumption Allowances, and the thin line as before includes labor income in mixed income. Data comes from OECD.

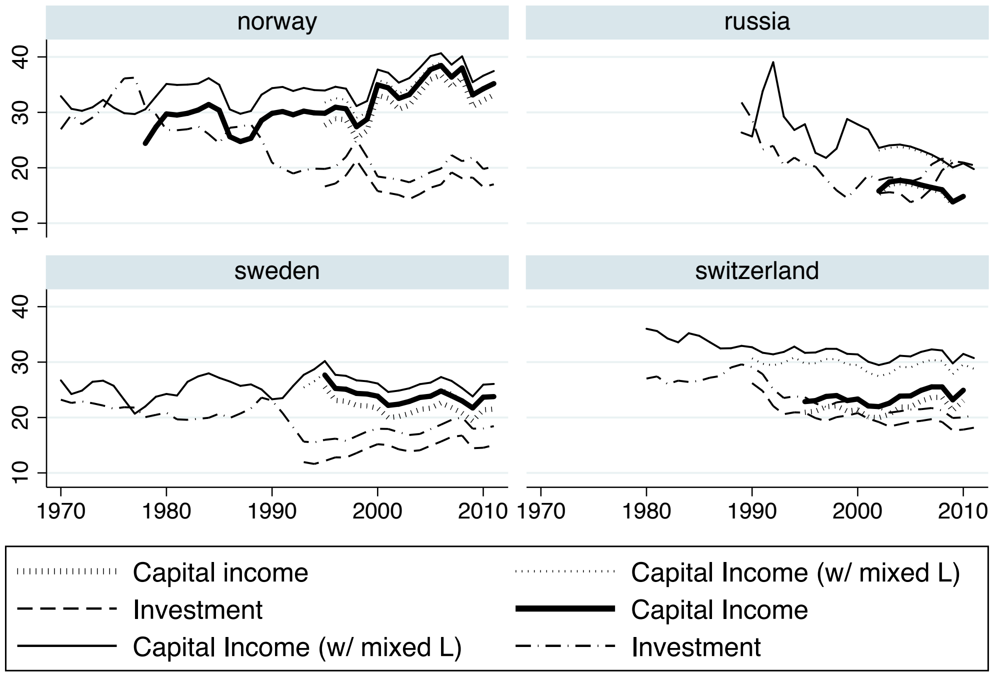
Figure 22: Assessing dynamic efficiency in France, Germany, Hungary and Italy



Source: OECD, Kuznets (1985), and author's calculations

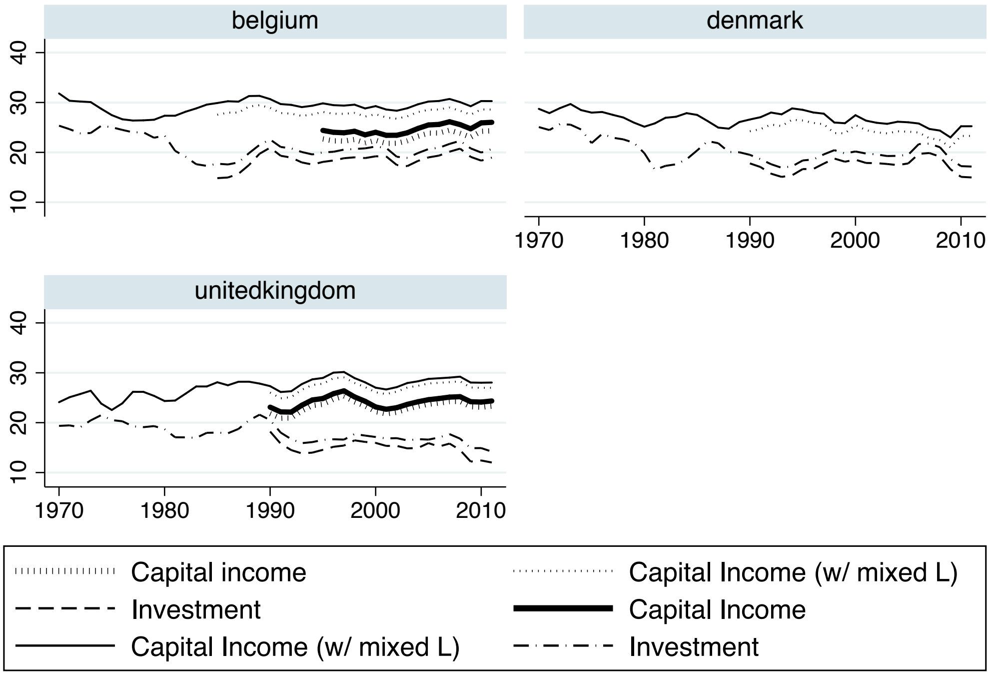
Notes: See Figure 21.

Figure 23: Assessing dynamic efficiency in Russia, Norway, Sweden, Switzerland



Source: OECD, Kuznets (1985), and author's calculations

Figure 24: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, DENMARK, UNITED KINGDOM



Source: OECD, Kuznets (1985), and author's calculations

Chapter 5

Dynamic Inefficiency and Capital Taxation

Abstract

Life cycle models of capital accumulation with land (or monopoly rents or decreasing returns) can only feature capital under accumulation, because of the extreme capital crowding out properties of capitalizable rents as one approaches the Golden Rule. In this paper, I show that it is no longer valid when a government levies property, wealth or estate/gift taxes. Contrary to infinite horizon models, positive capital taxes therefore help capital accumulation towards the Golden Rule in overlapping-generations models ; and no other set of transfers from young or old, public debt, or social security system can help achieve such a level of capital accumulation.

Keywords: Dynamic efficiency, land.

JEL classification: H55

Acknowledgements: I am deeply indebted to Guy Laroque for calling my attention to the dramatic capital crowding out properties of land when interest rates fall below the Golden-Rule level. I thank Emmanuel Farhi, Greg Mankiw, Philippe Martin, Thomas Piketty, Larry Summers and seminar participants at Harvard University, MIT and Paris School of Economics for comments and suggestions. I am extremely grateful to "Corps des Ponts" for funding this research and to Harvard University for their hospitality.

> "As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce. The wood of the forest, the grass of the field, and all the natural fruits of the earth, which, when land was in common, cost the labourer only the trouble of gathering them, come, even to him, to have an additional price fixed upon them. He must then pay for the licence to gather them; and must give up to the landlord a portion of what his labour either collects or produces. This portion, or, what comes to the same thing, the price of this portion, constitutes the rent of land ...." (Adam Smith, The Wealth of Nations)

Introduction

Capital taxes are usually viewed both in academic and policy circles as a deterrent to capital accumulation. Notwithstanding the fact that capital taxes distort savers' decisions, there is generally little arguing about the supply side effect of capital taxes : they lead to lower capital accumulation, and therefore lower output.[1] Even when capital taxes are not explicitely studied through the lens of capital accumulation, this wisdom is always somewhat present : among many other examples, New Dynamic Public Finance finds that positive capital taxes are optimal when agents are faced with idiosynchratic productivity shocks and governments can only tax in a distortive way, but that the positive welfare effects from those capital taxes are considerably diminished when one considers the general equilibrium effects taxation has on capital accumulation.[2]

This paper challenges this intuitive idea. In a nutshell, I recognize that savings do not only nourish capital accumulation but also serve to acquire non-reproducible assets such as land. Because of capitalization, this land is less valuable all the more that capital taxes are higher, so that capital taxes can actually help capital accumulation. To study the relative potency of these effects, I develop an Overlapping-Generations neoclassical growth model a la Diamond (1965) with an asset distributing rents each period (land rents are an example, but these rents can as well be monopoly rents, decreasing returns to scale as in Scheinkman (1980)). The dividends brought about by this asset endogenously grow at the rate of growth of output, which is meant to capture the idea that land is a constant (or growing) fraction of GDP; or, in the case of monopoly rents, that the economy does not become perfectly competitive asymptotically, which would happen if monopoly rents became negligible relative to GDP. As is very well known in this literature, the economy can then never reach an optimal level of capital accumulation corresponding to the Golden Rule.[3] The intuition is simple: land is a store of value for savings as capital, and therefore crowds out capital accumulation. As interest rates come closer to the Golden Rule, land becomes more and more valuable, infinitely valuable at the limit, thus making accumulation towards the Golden Rule level impossible. In this environment, capital taxes are a way to expropriate agents of these future dividends, thereby reducing the total supply of stores of value, and increasing the resources available for higher capital accumulation.

An important assumption maintained in all this paper is that governments cannot tax land (or monopoly or decreasing returns to scale) and productive capital separately: instead, they

[1] I voluntarily put aside here a discussion which was active in the eighties, about the fact that with an Elasticity of Intertemporal Substitution lower than 1 wealth effects could dominate over substitution effects, leading lower net of tax interest rates to increase savings, hence capital accumulation. However, I think that what comes out of this debate is that substitution effects dominate strongly once one takes into account realistic life cycle patterns for savings decisions. See for example Summers (1981). Moreover, there is a growing consensus that the inverse of the EIS and the coefficient of risk aversion - which is very high when backed out from the equity premium - are two different theoretical objects (Weil (1990), Epstein and Zin (1989)), so that the EIS is not so low.

[2] See Farhi and Werning (2012) for such a quantitative exploration. This idea also underlies Lucas (1990)' calculations that "supply-side economics", or removal of all capital taxes, would bring about.

[3] George J. Stigler recalls in his *Memoirs of an unregulated economist*: "Maurice Allais was a gifted engineer and economist, but at the time he believed that private ownership of land was untenable. (The reason need not occupy us; it turned on the fact that if the interest rate went to zero, as he feared it would, land would become infinitely valuable.)".

must tax both productive capital and rents are the same rate.[4] Rents and capital income are indeed not as easy to distinguish as in economists' models, because in practice, rents belong to the ownership of productive capital. For the purpose of taxing land rents for example (and them only), the government would need to charge homeowners a rent corresponding to the exact value of renting land, since land and residential structures are tied to one another. Needless to say, as the relative value of residential structures and land fluctuates over time[5], this would entail high administrative costs; the value of land could not be inferred from the resale value of a home either, as the value of home improvements (for example) is unobservable albeit to a very high administrative cost. In practice, the property tax therefore falls on both elements, hence, to quote Vickrey (1999),"The property tax is, economically speaking, a combination of one of the worst taxes, the part that is assessed on improvements and in some cases to a limited extent on personalty, and one of the best taxes, the tax on land or site value". [6] And although the model will only feature land rents, Scheinkman (1980) has shown that the crowding out properties of rents also apply to decreasing returns and monopoly rents, which are even harder to measure. Interestingly, this remark also suggests that one needs to go beyond Allais (1947)'s proposition that land should simply be nationalized: because land are not the only growing rents out there, all capital might in that case would need to be nationalized to the extreme. Adopting a market first, government second approach leads us to stay clear of such a proposition, and look for the lowest level of government ownership consistent with efficiency.[7]

Introducing capital taxation in a model with land rents allows to get several new results which were not known previously in the literature. A first result is that capital taxation has two opposite effects : it reduces the supply of savings for reasonable values of the Elasticity of Intertemporal Substitution[8], but it also decreases the demand for savings coming from land values, and competing with capital for savings. Second, a result in this literature is that even though capital taxation could be used to try and target an optimal level of capital accumulation[9], the government could use other tools to target the optimum level of capital: pay-as-you-go systems, or public debt. What comes out of this literature is that targeting the optimal level of capital accumulation does not provide a strong and convincing rationale for taxing capital. This paper shows in contrast that such schemes would not be sufficient to reach the Golden Rule, and that a strictly positive level of

[4]If this was not the case, then the government would optimally tax rents at a positive rate, but would not have to tax productive capital in the same way. Then, as in Ordover and Phelps (1979), capital taxation would be one of many instruments to target the optimal level of capital accumulation, together with public debt or assets and transfers to old and young generations.

[5]Economic geography and urban economics teach us that land values depend on the presence of a number of changing amenities.

[6]This point is discussed at length in the *Mirrlees Review* (IFS (2011)).

[7]For the sake of completeness, this would of course need to be justified in richer model. One can think that government planning requires much more information and understanding than it will ever have, and that a decentralized market economy is preferable to a planned one.

[8]As Summers (1981) in particular has shown, for realistic life cycle profiles one does not need a higher than 1 Elasticity of Intertemporal Substitution to get this result, because increased interest rates also reduces human wealth which increases the need for savings.

[9]See for example, Ordover and Phelps (1979). More in the following literature review. Positive capital taxation would be desirable in the case of too much capital accumulation, and the reverse in the case of too little. See Figure 2 for the inefficiency and Figure 3 for the efficiency case later in the text.

taxation is necessary to attain this social objective. The tax on capital making possible to reach the Golden Rule might be very small in practice (especially if further decreases in asset supply are made through an increase in public assets or in reverse pay-as-you-go systems); however the fact that capital *needs to be* taxed at a positive rate to favor capital accumulation is conceptually a very important result when the zero capital tax result is an important reference point both in academic and policy discussions. Moreover, high levels of public debt and of pay-as-you systems both rely on the possibility of commitment by the government, which it may be lacking in practice, to which taxes on capital can provide a good substitute.[10] Third, on the positive side, it shows that useful land and dynamic inefficiency (hence bubbles, Pareto-improving public debt, etc.) can coexist when some capital taxes are levied, a new result in the overlapping-generations literature. Fourth and perhaps more anecdotally, it allows to revisit other theoretical results which are taken for granted in the capital taxation literature, for example that taxes on flows and stocks of capital are equivalent.

Related Literature. The literature on life-cycle models and capital accumulation starts with Allais (1947) who remarks that there exists an optimal quantity of capital, or "Golden-Rule" level ("optimum capitalistique")[11] corresponding to a long run consumption-maximizing level of capital.[12] But as physical capital would be accumulating progressively, interest rates would go down and approach zero. This would drive the value of land to infinity, as it distributes constant dividends each period. Therefore, in order to get faster (and eventually) to the Golden-rule level of capital, Allais (1947) advocated complete nationalization of land, as George J. Stigler recalls in his memoirs quoted above.[13] This remark has later been used somewhat differently, to argue that dynamic inefficiency (capital accumulation above the Golden-rule level of capital) would be impossible, as it would make land impossible to transfer across generations.[14] Scheinkman (1980) has extended this argument to decreasing returns to scale technologies, which are another form of rent, to prove that economies with

[10]I have previously emphasized that the government could increase public assets or use reverse pay-as-you-go systems to try to mimic the imposition of a small capital tax. However, as I show in the rest of the paper, this effect on asset supply dominates for low values of the tax, while the effect on asset demand is more important when the value of the tax increases. The government will therefore want to use capital taxation to decrease asset demand if the small tax on capital has led to dynamic inefficiency.

[11]*En français dans le texte*: "Chaque jour l'ensemble de l'économie dispose d'un certain nombre d'heures de travail et la question se pose de savoir quelle est la répartition la meilleure qu'il y a lieu de faire de ce travail entre les différents stades de la production (biens directs et biens indirects)." Here is a tentative translation: "Every day an economy is given a certain number of hours of work, and the question is how we should use these hours as the different stages of production - direct or indirect." *Direct* production is the production of consumption goods and *indirect* production is the production of investment goods, which are later useful for producing consumption goods.

[12]For more on the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965).

[13]"En fait, il résulte de ce qui précède qu'une politique efficace d'annulation du taux de l'intérêt implique nécessairement comme conditions préalables la collectivisation de la propriété du sol et la dévalorisation de la monnaie circulante par rapport à l'unité de compte. Si de telles modifications de structure étaient apportées, non seulement l'annulation du taux d'intérêt deviendrait possible, mais encore on peut considérer que le taux d'intérêt qui tendrait à s'établir spontanément serait probablement nul, voire même légèrement négatif" (Allais (1947)).

[14]In fact, this argument is generalizable to a growing economy, no matter what the cause of this growth is (population or technological progress), as long as the share of land does not vanish relative to GDP. This is intuitive, as the Golden Rule level of capital accumulation verifies $r = f(k^{GR}) = n$ with n the rate of growth, and land dividends grow at rate n, so that when $r < n$, the value of land is again infinite.

a non-vanishing rent could not reach a dynamically inefficient state. Tirole (1985) examined this claim rigorously in a Diamond (1965) overlapping-generations growth model, and showed that rational bubbles could exist even when the economy wasn't asymptotically rentless. All that was needed was that rents be not capitalized ex-ante, so as not to be used by young generations as vehicles for savings.[15] However, as McCallum (1986) noted, land is a non-vanishing rent, capitalized ex-ante, hence ruling out the possibility of dynamic inefficiency and rational bubbles.[16] This problem was in fact also recognized as early as in Samuelson (1958), albeit somewhat indirectly.[17]

This paper also draws on the very large and developed literature dealing with capital taxation. The literature most directly linked to the present study is that concerning the link between capital taxation and accumulation. Ordover and Phelps (1979) showed that if the government had the policy tool to fix the capital stock at its optimal level, then the optimal capital tax rate should be zero, and Stiglitz (1985) gave a simpler proof of this same result. In the same line of thought, Atkinson and Sandmo (1980) show that whether capital taxation is of any use in this context depends a lot on the policy instruments of the government. This paper shows that with capitalizable rents, the government actually does not have any other tool to fix capital at its optimal level than to use strictly positive capital taxation. No level of finite public debt or of finite transfers from the young to the old can mimic the implementation of this outcome. By contrast, the supply-side calculations in both Lucas (1990) and Farhi and Werning (2012) (for example) take as a given that the capital stock is below the Golden Rule level, and that increasing the level of capital is welfare-enhancing. Moreover, because productive capital is the only asset in their models, the supply of stores of value is unaffected by the level of capital taxation. In contrast, in my model, the supply of stores of value decreases with higher capital taxes, which unambiguously mitigates the adverse effects on demand near the Golden Rule. Stiglitz (1978) shows that somewhat counterintuively, higher estate taxation can lead to higher inequality if the elasticity of capital and labor is less than 1 in the production function: in that case, higher capital taxation leading to lower capital accumulation can increase the factor share of capital which is more concentrated among high-income individuals. Once again, the negative effects of capital taxation on capital accumulation is here taken for granted. On the general subject of capital taxation, the literature puts forward other reasons to tax capital which I shall not consider here: unobservable wealth (Cremer et al. (2003)), indirect taxation of bequests and insurance of shocks on rates of returns (Piketty and Saez (2013)). In contrast, I will work under a paradigm of complete asset markets and perfect information, thus making the case for capital taxation even stronger. Compared to Piketty and Saez (2013), I do not leave the demand side effects of capital taxation as a free parameter: the elasticity of savings to the before tax rate are not infinite as with infinite horizon models, but they are endogenously determined (and

[15] For example, paintings, one type of rent, could not be sold by a painter's forebear.

[16] And, for his purposes, Pareto-improving money or other social contrivances. Rhee (1991) made the same argument and verified empirically that land rents were a non-decreasing share of GDP in the United States. Demange (2002) generalizes the argument to economies with uncertainty.

[17] "In it nothing kept. All ice melted, and so did all chocolates. (If non-depletable land existed, it must have been superabundant.)" (Samuelson (1958), p 481) Hence Paul Samuelson assumes that land is in infinite supply, so that it has no value.

finite) in models with finite lives.

The zero capital tax result remains a reference point in most academic work concerning capital taxation. Atkinson and Stiglitz (1976) show that if consumption is separable from leisure choices, and the economy deterministic, then savings decisions should not be distorted at the optimum. But they consider only life cycle savings, so that agents' preferences shall be respected: in contrast, both in Farhi and Werning (2010) and Piketty and Saez (2013), the social welfare function differs from individuals' objectives (for example, in Farhi and Werning (2012) the social planner puts more weight on future generations). Similarly, I will adopt a steady-state utility criterion a la Phelps (1965), so that individuals' decisions will not be optimal from the society's point of view. Moreover, I will consider a finite number of overlapping-generations, so that the first and second welfare theorems fail, while Atkinson and Stiglitz (1976)'s result relies crucially on the Pareto-optimality of the competitive equilibrium without taxes.Chamley (1986) and Judd (1985) show that Ramsey (linear) taxes on capital should be set to 0 in the long run, if individuals have infinite lives. Once again, this benchmark is one in which welfare theorems provide a reference point without taxes, and in which the economy is uniformly impatient. The utility criterion considered in this Chamley-Judd benchmark is implicitly the one used by the first generation to evaluate their children's utility. As is well known, such a perspective yields to the immiseration result, which is not very satisfactory from a welfare perspective.[18] Finally, capital mobility is another argument not to tax capital (see for example Gordon (1986)); though I will not consider capital mobility in the model, the results will go through with an higher elasticity of the supply of stores of value: because land is by definition immobile, the tax will not be completely shifted through lower accumulation.

Finally, it is important to understand that the research agenda carried out in this paper is somewhat orthogonal to the New Dynamic Public Finance literature (NDPF), though the latter also rationalizes strictly positive levels of capital taxation. I will consider only a deterministic environment, so that NDPF would prescribe zero capital taxation with my assumptions. Also, I will work with Ramsey (proportional) tax systems, in contrast to the NDPF which takes a Mirrlees (1971) approach to taxation. There are two reasons for this. The first one is that studying proportional taxes is a useful benchmark, on which some strong results are based (like Chamley (1986)-Judd (1985)) The second is that Atkinson and Stiglitz (1976) suggest that labor income and capital taxation are to a large extent orthogonal: mirrleesian taxation helps tax labor in the least distortive way, and capital taxes must be set to 0. In the case considered here, competitive equilibrium fails to be Pareto-optimal anyways and linear capital taxes are not necessarily distortive, but on the other hand can help restore efficiency.

The remainder of the paper proceeds as follows. Section 1 presents a Diamond (1965) model of overlapping generations with land rents. Section 2 gives the results. Section 3 discusses the limitations of the analysis and possible future work.

[18]In particular, Atkeson and Lucas (1992) show the importance of taking into account the welfare of children directly rather than only in the welfare of parents.

1 Model

In this section, I will develop a standard Diamond (1965) model with land. For concreteness, land will be a useful input in the production function, available in fixed supply. Note that this corresponds more to an "old" (agricultural) use of land (though businesses arguably need some amount of land to operate, also nowadays); a more "modern" use of land would correspond to an inclusion in the utility function, with two goods, a consumption good and a good corresponding to the utility for living in the business district:

$$U(C, L) = C^{1-\gamma} L^{\gamma} \quad \text{with} \quad L = 1.$$

The elasticity of substitution would be assumed to be one, so that the rental price of land would grow at the rate of growth of consumption g. Finally, the price of land could also be derived by recognizing the existence of some form of increasing returns (this could be done in an urban economics model - like the Alonso-Muth-Mills monocentric model for example).

1.1 Agents

There are overlapping-generations of agents. The generation born at time t consumes c_t^y when young and c_{t+1}^o when old. Work occurs only when young, and labor is supplied inelastically. Agents then earn a wage w_t, save s_t, on which they earn return r_{t+1}. They receive a transfer T_t^y from the government when young and T_t^o when old, so that:

$$\begin{cases} c_t^y + s_t = w_t + T_t^y \\ c_{t+1}^o = (1 + r_{t+1}) s_t + T_t^o \end{cases}$$

Therefore, their intertemporal budget constraint sums up to:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t + T_t^y + \frac{T_t^o}{1 + r_{t+1}} \tag{1}$$

They consume in both periods of their lives, and therefore they solve:

$$\max_{c_t^y, c_{t+1}^o} U(c_t^y, c_{t+1}^o)$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1 + r_{t+1}}$$

This gives a demand function for consumption when young and when old, and an implied savings demand, depending on wages and the interest rates. For simplicity, let us assume that the utility function exhibits Constant Relative Risk Aversion (CRRA) with risk aversion σ (and intertemporal elasticity of substitution $1/\sigma$):

$$U(c_t^y, c_{t+1}^o) = a \frac{(c_t^y)^{1-\sigma}}{1 - \sigma} + (1 - a) \frac{(c_{t+1}^o)^{1-\sigma}}{1 - \sigma}.$$

The solution of this problem yields the consumption decision of the young (see Appendix

1.1 for detail) :

$$c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma} (1+r)^{1/\sigma - 1}}.$$

And hence, savings are given by:

$$s_t = w_t + T_t^y - \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma} (1+r)^{1/\sigma - 1}}.$$

1.2 Production

Factor incomes are not exogenous. The supply of land is fixed to $L_t = 1$ for every t. On the production side, firms hire labor and use capital with a constant returns to scale technology with respect to joint labor and capital such that $Y_t = f(K_t, N_t, 1) = N_t F(k_t, 1, 1) = N_t f(k_t)$. Capital and labor earn their marginal returns expressed with the intensive form of the production function as:

$$\begin{cases} w_t = f(k_t) - k_t f'(k_t) \\ r_t = f'(k_t) \end{cases}$$

With a Cobb-Douglas production function, $F(K_t, N_t, L_t) = K_t^\alpha N_t^{1-\alpha} L_t^\beta$ and $f(k_t) = k_t^\alpha$, so that $w_t = (1-\alpha)k_t^\alpha$ and $r_t = \alpha k_t^{\alpha-1}$. Note that the rental price of land is also given by its marginal return, which gives the demand for land:

$$r_t^* = \frac{\partial Y_t}{\partial L_t} = \beta f(K_t, L_t, 1).$$

In equilibrium, r_t^* must be such that demand is consistent with supply $L_t = 1$. It is easy to see that capital will in that case receive r_t^* which increases like the rate of growth of output (and K_t and L_t on a steady-state growth path). Of course, the fact that land appears as having a unit elasticity of substitution with both labor and capital in the production function comes from the reverse-engineering of the share of land in total value added, which was first noted by Rhee (1991).[19] β parametrizes the constant relative importance of land with respect to other inputs. Except in the base where $\beta = 0$, land will turn out to play a very important role in the determination of asset supply.

[19]This was discussed more precisely in the introduction. The original argument, for example by Allais (1947), is formulated in an economy which does not grow. Hence, he considers constant dividends each period; and negative interest rates for dynamic inefficiency. shows using US data that land does not vanish relative to GDP, so that the same argument can be made with growth, g land rents growing at g, and interest rates below g for dynamic inefficiency. In contrast, Tirole (1985) rules out those growing with GDP capitalized rents by assumption, and only allows non-capitalized rents to grow at the same rate as GDP.

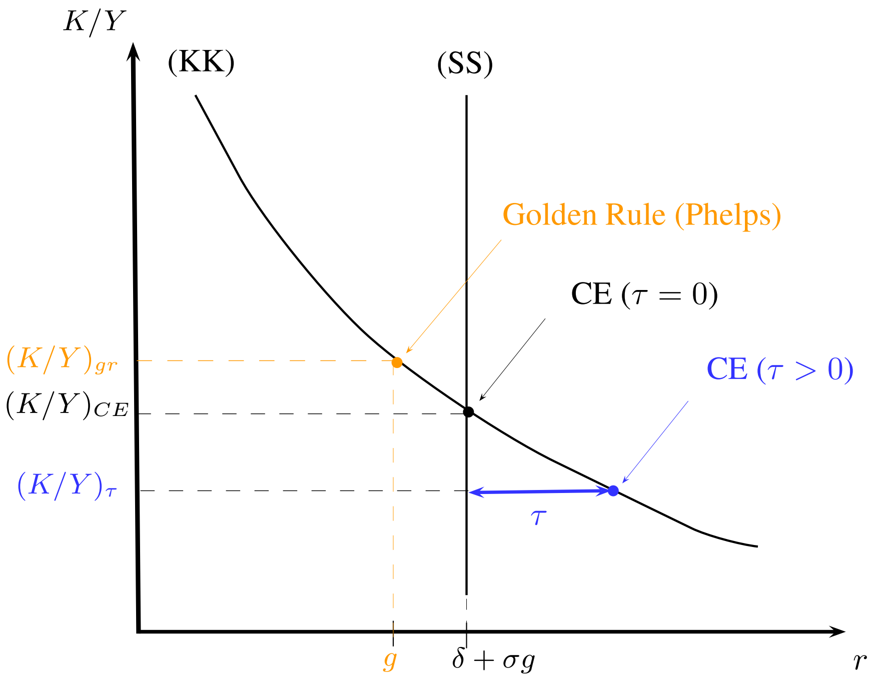
1.3 Discussion

Infinite horizon models. In the Fisher (1930) theory of interest, the after-tax interest rate is always equal to the so-called modified Golden Rule:

$$r = \delta + \sigma g.$$

This after-tax steady-state interest rate comes from a condition for agents' consumption at infinity; but the theory predicts very large variations of capital stock accumulation in response to capital taxes. In fact, the elasticity of savings to after tax interest rates is infinite in this model, as shown in Figure 1.

Figure 1: CAPITAL TAXATION IN FISHER (1930) THEORY OF INTEREST



Note: In Fisher (1930)'s theory of interest, the after tax rate of return is pinned down. Therefore, all the adjustment to tax rates goes through a reduction in the demand for capital by firms.

However, this theory is somewhat at odds with the data for at least two reasons. First, long-term interest rates move a lot over the business cycle (see for example Figure 5 for a plot of long term interest rates), thus explaining those movements would require very high "patience" shocks on the part of consumers (such shocks are indeed used in monetary models to generate movements in the natural rate of interest). Second, this theory predicts always higher than growth interest rates (in principle, the coefficient of relative risk aversion could be lower than 1, but it is both empirically implausible and theoretically problematic, as this would mean that the consumers' infinite horizon optimization program would not be well defined). Finally, as Piketty and Saez (2013) remark, it predicts very large movements of wealth/income ratios in relation with capital tax rates, which do not seem to be there in the data.

OLG models. In contrast, I use a finite life model, in which savings have a finite interest elasticity. Note that agents are in a sense inherently impatient in OLG models, as they don't care about after death consumption, and so their objective functions is always well-defined. At the same time, they are in many respects much less impatient than infinitely-lived optimizers, as they by assumption cannot borrow against future generations' income (they could not repay). For the finite live assumption to hold it is important that agents do not leave bequests a la Barro (1974); however they could well have some other form of dynastic altruism, like a warm-glow of giving bequests for example. The analysis naturally generalizes to this case.

2 Results

2.1 Some heuristics

No land

As is well known, with no land the competitive outcome hence will not correspond to the steady-state maximizing outcome, albeit for very particular values of the parameters. When there is capital under-accumulation, one might need to reduce some generations' consumption in order to improve future generations' welfare, and reach the Golden-rule level of steady-state consumption (this is likely to be the rationale behind savings-enhancing policies). However, under capital over-accumulation, that is $k^* > k^g$ there are allocations that unambiguously improve on the welfare of all generations. If $k^* - k^g$ is added to consumption when steady state has been reached[20], then consumption in period T is

$$\begin{aligned} f(k^*) + k^* - k^g - nk^g = f(k^*) &- nk^* + (n+1)(k^* - k^g) \\ &> f(k^*) - nk^* \end{aligned}$$

Subsequently quantity available for consumption is

$$f(k^g) - nk^g > f(k^*) - nk^*.$$

This case therefore leads to a well known result: dynamic efficiency or inefficiency depends upon the parameters of the economy. The economy can by itself accumulate too much capital or too little. However, as Ordover and Phelps (1979) note for example, there is no clear role for capital taxation in this model as pay as you go systems and public debt can as well help the economy reach the Golden Rule of capital accumulation.

[20] It might be that this level is never reached, but only asymptotically. Yet the argument remains considering a large enough T above which one lies arbitrarily close to steady-state values.

Land with no taxes

Land substantially change the analysis of the previous model. Denote by R the steady state share of land in GDP (corresponding to the limit of r_t^*/Y_t as t approaches ∞). Then land distributes $\frac{R(1+n)^t}{1+r}$ in period t, and its value at time t hence is $\frac{R}{r-n}(1+n)^t$ or $R/(r-n)$ per capita.[21] Note that at date $t+1$, each generation born in period t gets a value of the rent per capita that decomposes in the following way:

$$\frac{R}{r-n}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n}}^{\text{capital gains}}}_{\text{total return}}.$$

I then state the following proposition, generalizing Allais (1947)'s fears that an economy with land will never reach the Golden Rule level of capital accumulation, let alone dynamic inefficiency:

Proposition 7. *Whenever land remains productive (that is $r_t^*/Y_t \to R > 0$), the competitive allocation always displays under-accumulation relative to Golden-Rule level of capital, that is $k^* < k^g$. This is true no matter what the steady-state levels of transfers T_o or T_y between old and young, or finite levels of government debt (positive or negative).*

Proof. Let $(b, T^y, T^o) \in \mathbb{R}^3$ per-capita public debt, per-capita transfers to the old and per-capita transfers to the young such that

$$T^y + \frac{T^o}{1+n} = 0.$$

For levels of public debt per capita b ($b < 0$ denotes assets):

$$(1+n)k^* + \frac{R}{\alpha(k^*)^{\alpha-1} - n} = (1-a)(1-\alpha)(k^*)^\alpha + (1-a)T^{y'} - \frac{aT^{o'}}{1+\alpha(k^*)^{\alpha-1}}$$

with:

$$\begin{cases} T^{y'} = T^y - b(1+r) \\ T^{o'} = T^o + (1+r)(1+n)b \end{cases}$$

Hence, Golden-rule or above Golden-rule capital accumulation are impossible, no matter what the values of b, T^y, and T^o are. □

[21] The value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R(1+n)^i}{(1+r)^{i+1-t}} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{1+n}{1+r}\right)^i = \frac{R(1+n)^t}{r-n}.$$

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Land with taxes

For simplicity, let us assume that the government cannot tax land and productive capital differently, as discussed in the Introduction. In that case, the value of land at date t is

$$p_t = \sum_{i=t}^{+\infty} \frac{R}{1+r}(1+n)^i \left(\frac{1-\tau}{1+r}\right)^{i-t} = \frac{R(1+n)^t}{1+r} \sum_{i=0}^{+\infty} \left(\frac{(1+n)(1-\tau)}{1+r}\right)^i = \frac{R(1+n)^t}{r-n(1-\tau)+\tau}.$$

This time at $t+1$, each generation born in period t gets a value of the rent per capita that yields return r by arbitrage decomposing in the following way:

$$\frac{R}{r-n(1-\tau)+\tau}(1+r) = \underbrace{\overbrace{R}^{\text{land dividend}} + \overbrace{\frac{R(1+n)}{r-n(1-\tau)+\tau}}^{\text{capital gains}} - \overbrace{\frac{R\tau(1+n)}{r-n(1-\tau)+\tau}}^{\text{wealth tax}}}_{\text{total (net of tax) return}}.$$

Proposition 8. *With positive wealth taxes $\tau > 0$, there exist values for $(a, \alpha) \in [0, 1]$ and $R \in \mathbb{R}_+^*$ such that the economy is inefficient or $k^* > k^g$.*

The intuition is straightforward: because the condition for finiteness of land values is now $r - n + \tau(1+n) > 0$, or $\alpha(k^*)^{\alpha-1} - n + \tau(1+n) > 0$, which does not rule out that $k^* > k^g$.

2.2 Quantitative results

No land

Let me assume for the moment that land is not valuable in production, that is: $\beta = 0$. It that case:

$$\forall t > 0, r_t^* = 0.$$

Then, at the steady state, the (SS) curve writes:

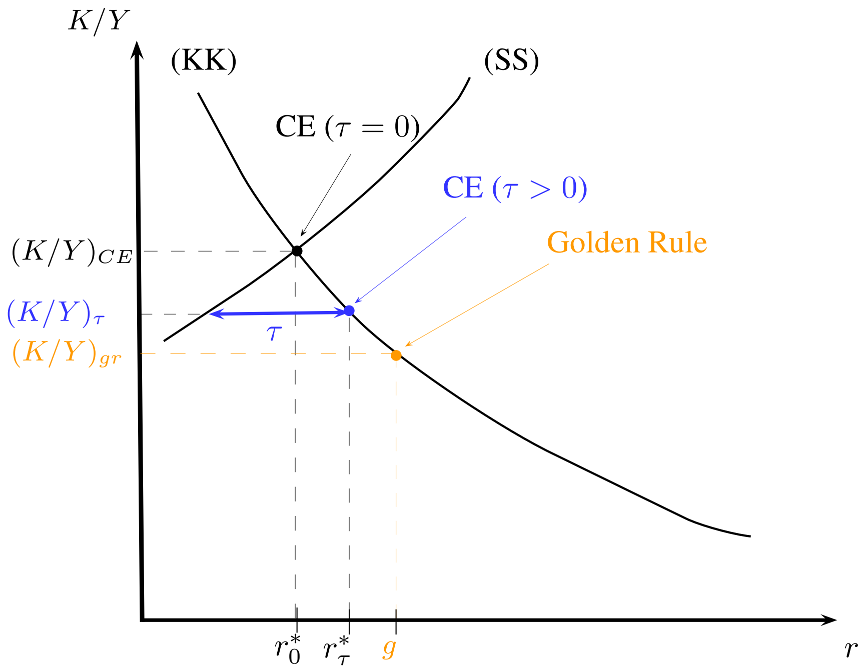
$$ss(r,0) = (1-\alpha)\left(1 - \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Under the assumptions outlined above, this defines an upward sloping locus in the $(r, K/Y)$ plane. That is, the assumptions are equivalent to assuming that:

$$ss'(r,0) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}} \frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}} \frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} > 0.$$

The (KK) curve is standard, and defines a downward sloping curve in the $(r, K/Y)$ plane. As is well known in this case, there can be dynamic inefficiency like in the case depicted in Figure 2 or dynamic efficiency like in Figure 3.

Figure 2: OLG model - Dynamic Inefficiency



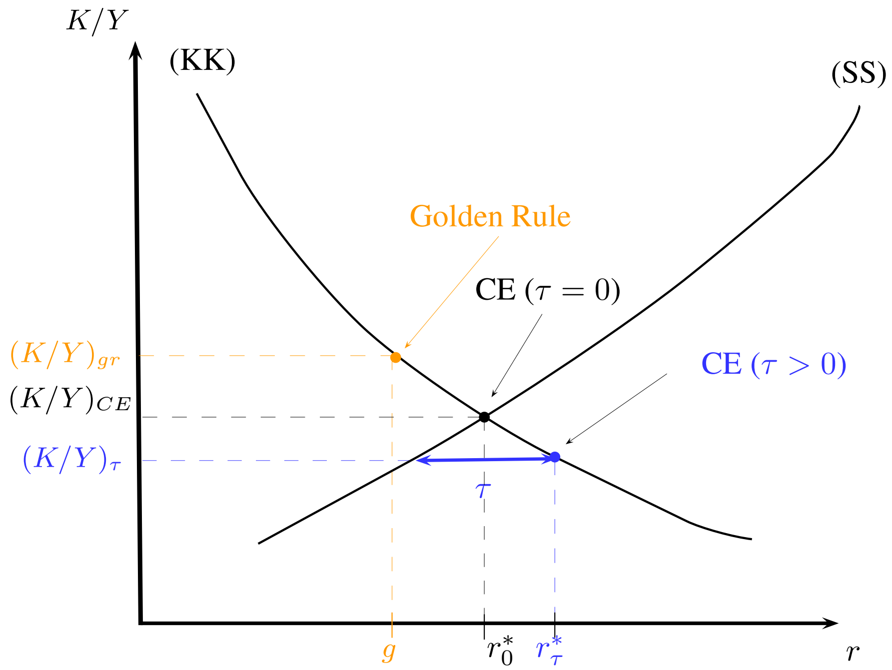
Note: Steady-state of an OLG neoclassical growth model without land. As Diamond (1965) showed, the competitive equilibrium may feature too much capital accumulation. In that case, capital taxes can help discourage capital accumulation and hence help reach the Golden Rule. However, as Ordover and Phelps (1979) have shown, public debt or pay-as-you-go systems can as well be used for this matter.

The effects of capital taxation on capital accumulation are unambiguous on those two figures. If the competitive equilibrium of the undistorted economy leads to too much capital accumulation as on Figure 2, then capital taxation allows to get closer to the Golden-Rule. Denoting by $(K/Y)_{gr}$ the capital/output ratio at the Golden Rule, $(K/Y)_{\tau}$ the same ratio at the distorted allocation (with capital taxed at rate τ), and $(K/Y)_{CE}$ the undistorted capital/output ratio, it is clear that:

$$(K/Y)_{gr} < (K/Y)_{\tau} < (K/Y)_{CE},$$

so that imposing a capital tax leads to an unambiguously better outcome (note however that this holds for a small enough value for τ). However, as Ordover and Phelps (1979) suggested, the government could just as well target a higher level of public debt or put in place pay-as-you go systems to remedy this dynamic inefficiency problem. Conversely, the case with dynamic efficiency depicted on Figure 3 would suffer from capital taxation, as the resulting capital/output ratio would be even lower. With the same notations, it is clear from the figure or from basic algebra that:

Figure 3: OLG MODEL - DYNAMIC EFFICIENCY



Note: Steady-state of an OLG neoclassical growth model without land. Here the competitive equilibrium is efficient. As with the Fisher theory of interest, capital taxation discourages capital accumulation away from the Golden Rule. Note that this effect is however dampened by the fact that the elasticity of savings to the rate of interest is finite.

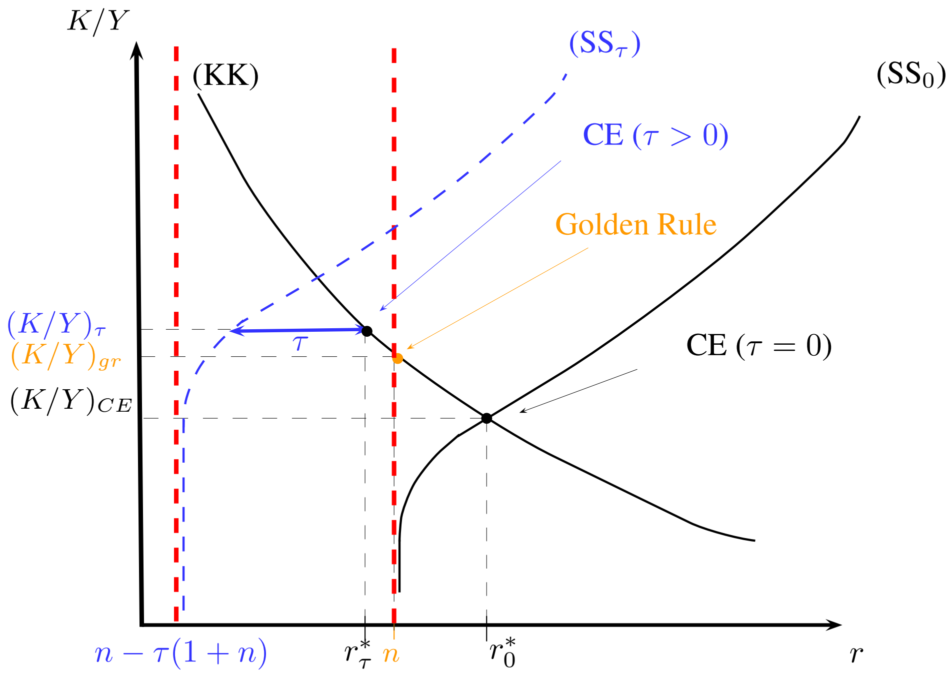
$$(K/Y)_\tau < (K/Y)_{CE} < (K/Y)_{gr}.$$

With land

In contrast, in the more realistic case where $R \neq 0$, there are two opposing forces. On the one hand, holding land values constant, capital taxes also have the effect of driving a wedge between the return earned by savers and that paid by entrepreneurs, decreasing overall capital accumulation. However, capital taxes also lead to a decrease in land values, thus increasing the amount of resources available from savers to productive capital (which I call "free savings" in the following).

These two opposing forces are visible on Figure 4. The latter effect is shown as a leftward shift in the $(r, K/Y)$ plane of the free-savings curve from (SS_0) to (SS_τ): with increases in capital taxation, the resources available for productive capital K increase for a given level of interest rates. Note how initially, because of the extreme capital crowding out properties of land when the interest rate approaches the Golden Rule, the curve (SS_0) was always to the right of the $r = n$ curve. In contrast, with positive levels of capital taxation, the curve (SS_τ) is totally consistent with a Golden-Rule level of capital accumulation.

Figure 4: OLG model - With land



Note: Steady-state of an OLG neoclassical growth model, incorporating land rents. Capital taxation has two effects on capital accumulation. First, it increases the supply of free savings for capital accumulation, by reducing land values. The curve (SS) shifts from (SS)$_0$ to (SS)$_\tau$. Second, it discourages capital accumulation by driving a wage between the rate of return for investors and the rate of return for savers. In the case pictured on this graph, the net effect of a capital tax on capital accumulation is positive.

Algebraically, free savings (that is, savings available for productive capital investment) are equal to:

$$ss(r, R) = (1-\alpha)\left(1 - \frac{1}{1 + \left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma - 1}}\right)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{R}{r - n(1-\tau) + \tau}.$$

Note that this "free savings" curve is steeper than the previous one without land. Under the previous assumptions, the (SS) curve is therefore unambiguously increasing in the $(r, K/Y)$ plane. This is because:

$$\begin{aligned}\frac{\partial ss(r,R)}{\partial r} &= (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma}}\frac{(1+r)^{\frac{1}{\sigma}-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} + \frac{R}{(r-n(1-\tau)+\tau)^2}\\ &= \frac{\partial ss(r,0)}{\partial r} + \frac{R}{(r-n(1-\tau)+\tau)^2} > \frac{\partial ss(r,0)}{\partial r}.\end{aligned}$$

The fact that $(SS)_0$ is always to the right of the $r = n$ vertical line, that a positive level of capital taxation corresponds to a leftward shift in the curve, etc. follows immediately from the examination of these functions. In the case shown in Figure 4, we have that:

$$(K/Y)_{gr} < (K/Y)_{CE} < (K/Y)_{\tau}.$$

In that case, positive capital taxation allows for dynamic inefficiency.

3 Discussion

Before making a few side remarks on the model, let me give a fairly simple intuition for why increasing capital taxes can lead to higher capital accumulation. In fact, committing to taxing land in the future amounts for the government to take on a (possibly very large) positive asset position. Just as public debt crowds out capital accumulation, public assets encourages it. And since Ricardian equivalence does not always hold in overlapping-generations models, this very high increase in government savings is not necessarily matched by a corresponding decrease in private savings. As was the case on Figure 4, it can even be that a sufficient amount of stores of value is no longer available once the capital tax has been imposed, so that the economy is dynamically inefficient.

3.1 Non-equivalence between stock and flow taxes on capital

A side result mentioned in the introduction is that in this framework wealth taxes and dividend taxes are not equivalent. In the model, very different would be to impose a positive capital income tax τ, which would tax dividend uniformly across time, and effectively expropriate private agents of a fraction τ of their capital stock (thus, previous arguments, which require only an arbitrarily small yet positive amount of land, would still hold). Therefore, allowing for a pre-existing factor of production like land allows to break down a well-known equivalence result between capital income taxation and wealth taxation. This result only holds *in equilibrium*, as taxation on capital τ_K and taxation on wealth τ_W are linked by :

$$1-\tau_W)(1+R) = 1+(1-\tau_K)R \quad \Leftrightarrow \quad \tau_K = \frac{\tau_W(1+R)}{R}.$$

3.2 Government's liquidity and solvency

A side result from the theory developed above is that if there is dynamic inefficiency because the government uses capital taxation (on land in particular), then governments are solvent since they potentially have infinite assets. These assets could be used to back potentially very large levels of public debt. However, there are several reasons why those infinite positive assets cannot be used to obtain financing. A first solution would be to sell those assets, but it is not possible since the government does not legally own future land rents. It only has the right to levy a tax on the value of this land in the future, and the right to collect taxes is not transferable to private agents. This asset therefore is not redeployable, and therefore cannot be sold nor directly collateralized. A second, more intermediate solution would be to promise the revenue from wealth or property taxes to new bondholders. However, this overlooks that property taxes, which represent the bulk of land taxes in most countries, are usually levied by local governments.[22]

Conclusion

This paper is only a first attempt at incorporating land and monopoly rents into a life cycle theory of capital accumulation. It has shown that the relationship between capital accumulation and taxation is far from one-directional in such an environment. Some positive level of capital taxes is even necessary to achieve the Golden Rule level.

There are many other elements which have been left out here, but which could be fruitfully integrated in future work. In this analysis, the government did not have any financing needs on its own. Needless to say, the introduction of a need for financing public expenditures would likely strengthen the case for capital taxation since land and monopoly rents represent pure profits. The alternative would be to tax labor income, which is distortive with flexible labor supply, even when it is done in a Mirrleesian way.

If the zero capital tax benchmark is not a reference point, then this paper naturally calls for quantitative evaluation of the effects it presents. In particular, Figures 6 and 7 show that there is a high cross-sectional variation of wealth, property and bequest/gift taxes over time, which might be fruitfully used for empirical analysis.

[22] And in effect, such public debt financing schemes has already been used by the city of San José in the United States to finance municipal debt. As Michael Lewis (2013) anecdotally puts it: "It is one of the few cities in America with a triple-A rating from Moody's and Standard & Poor's, but only because its bondholders have the power to comel the city to levy a tax on property owners to pay off the bonds."

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1 Omitted proofs

1.1 Agents' optimization

Agents solve the following two period standard optimization problem:

$$\max_{c_t^y, c_{t+1}^o} a\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + (1-a)\frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma}$$
$$\text{s.t. } c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \leq w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}.$$

Eliminating the multiplier on the budget constraint gives:

$$\left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma = (1+r_{t+1})\frac{1-a}{a}.$$

Replacing the expression for c_{t+1}^o in the resource constraint gives:

$$\left[1 + \left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right]c_t^y = w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}} \quad \Rightarrow \quad c_t^y = \frac{w_t + T_t^y + \frac{T_t^o}{1+r_{t+1}}}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}$$

1.2 Monotonicity of the steady-state "free savings" function

Denoting by $ss(.)$ the function associating free savings to the equilibrium interest rate,

$$ss(r) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1-a}{a}\right)^{1/\sigma}\frac{(1+r)^{1/\sigma-1}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}}.$$

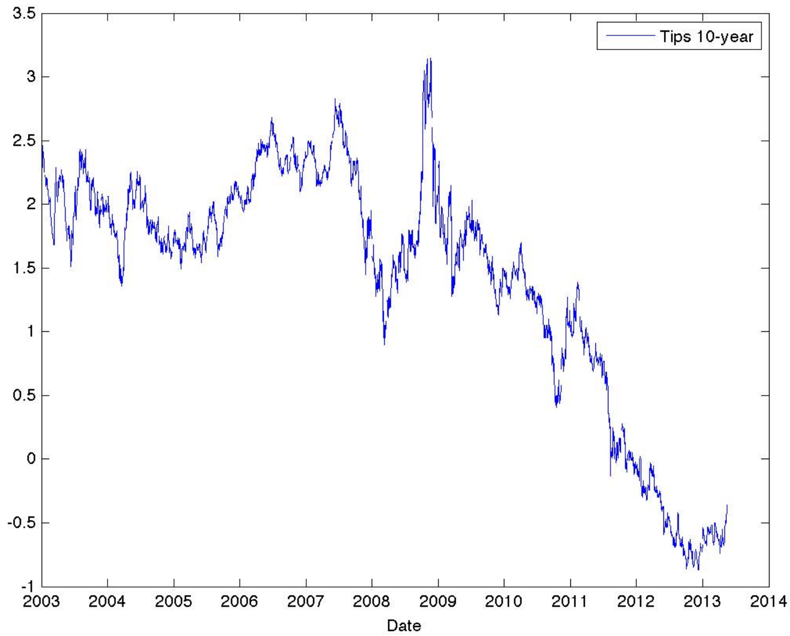
It follows that $ss(.)$ is non monotone since:

$$ss'(r) = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(1+r)^{1/\sigma-1}\left(\frac{1-a}{a}\right)^{1/\sigma}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}\left(1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}\right)}\left[\left(\frac{1}{\sigma}-1\right)\frac{1}{1+\left(\frac{1-a}{a}\right)^{1/\sigma}(1+r)^{1/\sigma-1}} - \frac{\alpha}{1-\alpha}\frac{1+r}{r+\delta}\right],$$

but for $1/\sigma - 1$ large enough, and low enough values for r it will be monotonically increasing. Note that this assumption is akin to assuming that increases in interest rates do not have too much adverse impacts on wages which end up to depress savings. A similar assumption is actually made in Diamond (1965) where interest rates also appear on both sides of an equation and some single-crossing and stability assumptions must be made.

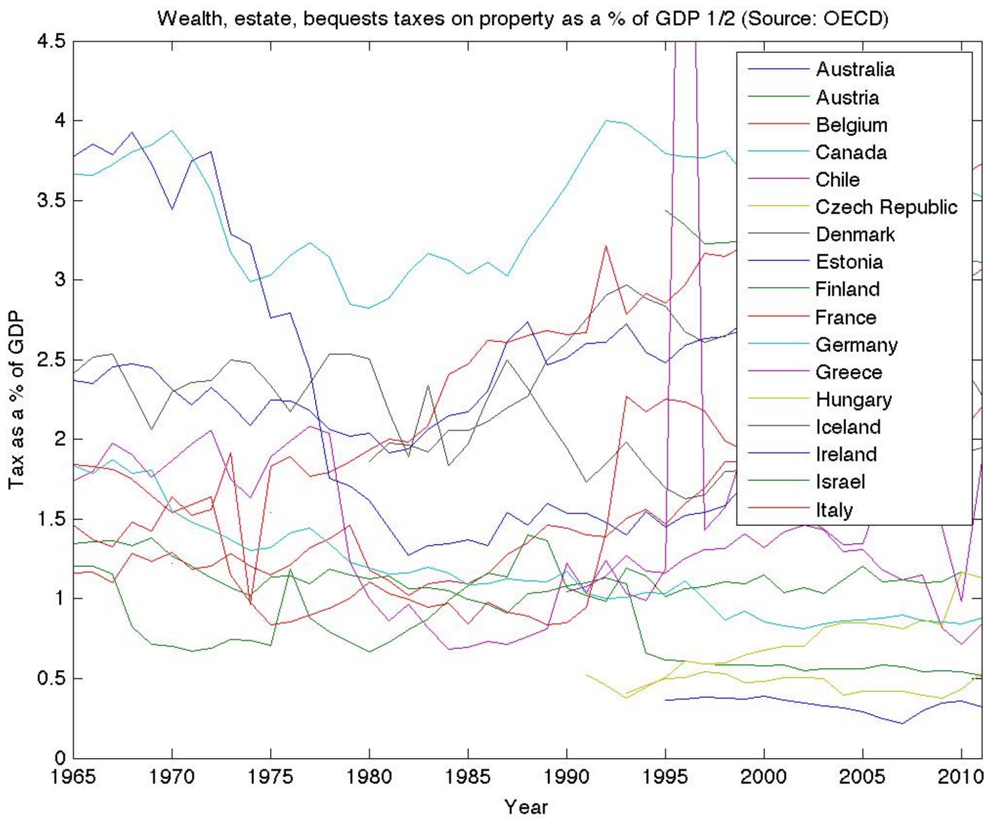
2 Figures

Figure 5: TREASURY INFLATION PROTECTED SECURITIES 10-YEAR



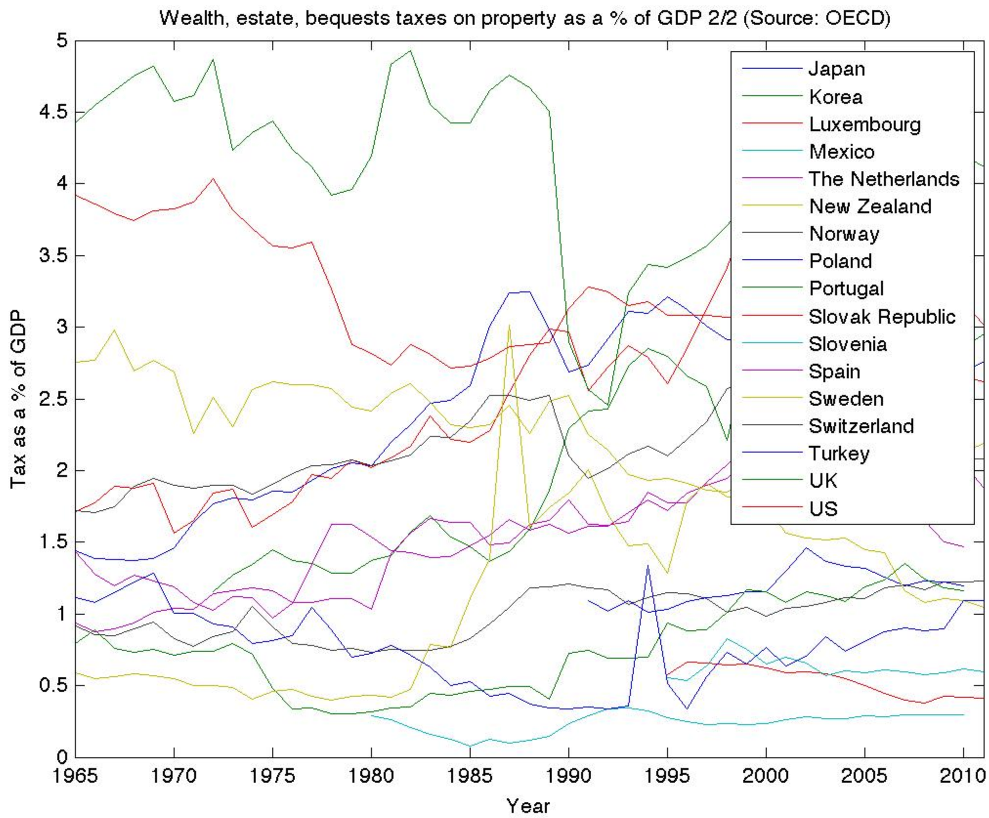
Note: Yield on Treasury Inflation Protected Securities (TIPS) with a maturity of 10 years. These are a proxy for expected real returns on assets. TIPS do not have much of a liquidity value.

Figure 6: PROPERTY TAXES AROUND THE WORLD 1/2



Note: The data is for Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Israel, Italy. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.

Figure 7: Property taxes around the world 2/2



Note: The data is for Japan, Korea, Luxembourg, Mexico, The Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. This figure shows that property, wealth and bequests/gifts taxes are always present in the world. This suggests that nowhere can land or monopoly rents prevent accumulation of capital towards the Golden Rule.

Conclusion

At the end of the journey, I hope to have convinced the reader that both empirically and theoretically, allowing for rational bubbles as a solution of the traditional asset pricing equation can help shed a new light on otherwise hard to understand stylized facts. This thesis was just a sketch of the myriad of possibilities opened by rational speculative bubbles, but there is a lot more work needing to be done. In financial economics, the equity premium puzzle[23] could well (at least in part) be a "maturity premium": because equities are infinitely lived assets, they could well carry a bubble component that would explain their higher relative performance. Bubbles could also potentially explain the ways of booms and busts in mergers and acquisitions that vary a lot around the business cycle.[24]

Similarly, many puzzles in international macroeconomics could potentially be studied under the light of "bubbles and asset supply". The exchange rate disconnect puzzle (Meese and Rogoff (1983)), as well as the forward premium puzzle (Fama (1984)), could be the mirror image of bubbles on other asset classes. Similarly, Chapter 4 only suggests a sketch of what International Real Business Cycle models could be; in particular, extending Chapter 3 to a two-country model could help explain many well known International Business Cycle puzzles. Among other examples, bubble shocks drive consumption booms at the same time as they drive real exchange rate appreciations, which could shed a new light on the Backus and Smith (1993) puzzle. Similarly, consumption is more correlated across countries than output because consumption booms are associated with decreases in net exports, again as a result of rational bubble shocks as shown in Chapter 4. Home bias is perhaps not so surprising if risk comes from multiple equilibria, and there is even a very small cost of trading assets across borders. In the same line of thought, it is still a puzzle why financial liberalization does not come about with higher benefits when the neoclassical assumption of diminishing returns to capital says it should help allocate capital to its most productive uses (Gourinchas and Jeanne (2006)). But if the world has indeed accumulated enough capital, this should not come as a surprise. In that case, capital account openness allows bubbles to be financed internationally, rather than only locally, as evidence in Chapter 4 seems to suggest. Countries would be all the more capable of withstanding these rational bubble shocks that they have more liberalized goods markets, which makes current account adjustments less costly (Martin and Rey (2006)).

[23]The so-called "equity premium puzzle" is the fact that equity returns have exceeded returns on bonds (say 3-months commercial paper) by a very high margin, much more than would be implied by their relative riskiness, the Consumption Capital Asset Pricing Model, and plausible values for risk aversions (that is, lower than 20).

[24]And indeed, anecdotal evidence suggests that funds sometimes buy non-listed firms in the hope of selling them back a few years later to a "greater fool".

In this thesis, and in particular in Chapter 2, I have tried to refrain from using sunspots as exogenous events that coordinate agents around the same equilibria (and that occur with some probability that every agent supposedly knows). Sunspots are a way of reintroducing measurability in a world where a comparative statics exercise is otherwise thought to be impossible. In Chapter 2, I have however tried to do equilibrium selection somewhat differently, by alluding to adaptive expectations as a coordination device. In this world, there is no notion of measurability which can create a high demand for hedging. Philippon and Reshef (2012) document a growth in the share of finance in GDP over in the United States (and for the world in Philippon and Reshef (2013)), which was similarly a feature of the US economy before the Great Depression in 1929. The dynamic inefficiency view implicitely provides a skeptical view of such a growth. In a world plagued by multiple equilibria on asset prices, volatility is excessive and demand for managing these risks grow. Getting rid of rational bubbles (and the associated excess savings problem) would likely dampen the volatility that generates profits in many trading activities. Very importantly, this "excess volatility" is not just financial as Chapter 4 suggests : it also translates into some "excess volatility" for the "real" sector as well. For example, the rise and fall of rational bubbles on national real estate markets do lead to an appreciation in the real exchange rate of these countries - and it does not matter whether this adjustment goes through the nominal exchange rates or through prices. In any case, these incoming capital flows enter in competition with exports to pay for imports (as in the usual "Dutch disease" phenomenon, which is a problem both for countries receiving development aid, and commodity exporters). Real exchange rates, whose variations are costly to tradable firms as is apparent in policy debates, must be hedged but not out of fundamentals' instability, but because of a mass of capital looking for stores of value.

If this approach is to be followed, then the world is to a large extent characterized by *too much* fluctuations. A natural extension of Chapters 3 and 4 would be to study the implications on unemployment of such capital movements. Over and above the desincentives to work brought about by the collapse of a rational asset pricing bubble (see Chapter 3), a displacement effect from tradables to non-tradables occurs in a boom characterized by large capital inflows, which must be scaled down during the bust. The resulting unemployment is to a large extent, mismatch unemployment. This mismatch is likely to be compounded by the fact that bubbles lead to a redistribution of resources from younger generations to old generations, and from poorer families to wealthier families in the case of bequests, whose consumption baskets are very different. After bubbles' bust, firms face huge uncertainties because they do not know when and how fast the next transfers of purchasing power will occur, which is likely to slow investment and hiring further. I leave this fascinating topic to future research.

I also hope to have convinced the reader that it is not necessary to adhere fully to the assumptions of price stickiness to find a role for Keynesian-like policies: monetary policy interventions, expanding of the money supply, countercyclical budget deficits. The dynamic inefficiency view of the world is able to generate mismatch unemployment about which the government could do something both ex post and ex ante. Ex-post, it could try to reflate the bubble, do countercyclical fiscal policy to allow the displacement effect from tradables to non tradables to come down only gradually. Ex-ante, it could use capital controls, capital requirements, loan to value ratios or other types of so-called macroprudential tools, as shown in Chapter 2. An interesting yet unanswered question is what public authorities should do in the long run to address the dynamic inefficiency problem and avoid future financial crises. Should they rather try and stabilize private bubbles on several asset classes, or increase public debt? This is already on next generation's research agenda.

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