Macroeconomic Theory 102
Winter 2015 - François Geerolf
Final Exam
Wednesday, March 18, 2015
Time Limit: 180 Minutes

Last Name: ____________________

First Name: ____________________

Teaching Assistant: ____________________

Student ID Number: __________

Signature ____________________

Test A

The Final Exam contains 26 pages (including this cover page). You can earn 100 points, and 3 bonus points for the very last question.

Instructions:

1. Print your Last name, First Name, Teaching Assistant Name (as a reminder, teaching assistants are: Flavien Moreau, Keyyong Park, Matias Vieyra, and Gabriel Zaourak), Student ID Number and Signature at the top of this page.
2. The only items which should be on your desk are pencils and/or pens. NO other items are allowed. Place any other item UNDER your desk. Calculators are NOT allowed.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck ! Budget your time wisely ! (skip the question or even the exercice if you get stuck)

Grade Table (FOR TEACHER USE ONLY)

Question	Points	Score
1	20	
2	12	
3	18	
4	7	
5	8	
6	15	
7	23	
Total:	103	

1 Multiple Choice Questions (20 points)

1. Mark box if true - each multiple choice question has only one correct answer.

   (a) (1 point) In the Solow model, with population growth:
   - ◯ there is no steady state in output per person
   - ◯ the economy never settles down to a steady state and exhibits growth in output per person
   - √ **the economy eventually settles down to a steady state in output per person**
   - ◯ the economy eventually settles down to a steady state with no growth in aggregate output
   - ◯ None of these answers are correct.

   (b) (1 point) In the combined Solow-Romer model, the total output growth rate:
   - ◯ equals the growth rate of ideas.
   - √ **is greater than the growth rate of ideas.**
   - ◯ is lower than the growth rate of ideas
   - ◯ equals the rate of capital depreciation
   - ◯ equals the growth in labor productivity.

   (c) (1 point) The Great Depression stimulated ________ to write ________, which is considered to be the birth of modern macroeconomics.
   - ◯ John Hicks; *Value and Capital*
   - ◯ Karl Marx; *Das Kapital*
   - ◯ David Ricardo; *Principles of Political Economy and Taxation*
   - ◯ Thomas Piketty; *Capital in the 21st century*
   - √ **John Maynard Keynes; *The General Theory of Employment, Interest, and Money***

   (d) (1 point) Suppose you put \$100 dollars in the bank on January 1, 2015. If the annual nominal interest rate is 2 percent, and the inflation rate is 5 percent, you will be able to buy ______ worth of goods on January 1, 2016.
   - ◯ \$95
   - ◯ \$102
   - √ **\$97**
   - ◯ \$103
   - ◯ -\$3.

   (e) (1 point) Compared to the nominal interest rate, the real interest rate is:
   - ◯ negative
   - ◯ always smaller
   - ◯ always greater than zero

√ **relatively stable**
◯ relatively volatile.

(f) (1 point) The costs associated with changing prices in times of inflation are called:
◯ inflation risks
◯ price staggering
◯ transaction costs
√ **shoe-leather costs**
◯ menu costs.

(g) (1 point) When we add the financial friction to the AD curve it:
◯ is represented by a downward movement along the AD curve
◯ is represented by a downward movement along the AS curve
√ **shifts the AD curve down**
◯ shifts the AS curve up
◯ has no impact on the AD curve.

(h) (1 point) "DSGE" stands for:
√ **dynamic stochastic general equilibrium**
◯ deterministic simulated generalized estimation
◯ demand supply generated effects
◯ discrete stationary generated equilibrium
◯ demand and supply government expenditures.

(i) (1 point) Suppose a worker gets a new computer; this would:
◯ decrease the nominal wage
◯ increase total factor productivity
◯ increase the amount of leisure demanded
√ **increase the marginal product of labor**
◯ None of these answers are correct

(j) (1 point) Using the labor market in the stylized DSGE model, from workers' perspectives, which of the following would be beneficial to them? (i) a reduction in the sales tax rate, (ii) a reduction in the income tax rate, (iii) a reduction in TFP.
◯ (ii)
◯ (ii) and (iii)
◯ (iii)
◯ (i), (ii) and (iii)
√ **(i) and (ii)**

(k) (1 point) If the growth rate of the capital stock is 9 percent, the user cost of capital is 13 percent, the capital depreciation rate is 3 percent, and capital gain is 1 percent, then the investment rate is about:

- ◯ 6.6 percent
- √ **30.8 percent**
- ◯ 11.7 percent
- ◯ 1.4 percent
- ◯ 50 percent

(l) (1 point) The investment-GDP ratio will rise if:

- ◯ depreciation falls.
- √ **user cost falls.**
- ◯ capital growth is negative.
- ◯ the marginal product of labor is positive.
- ◯ stock prices fall.

(m) (1 point) When capital depreciation is included in the arbitrage equation for capital, the user cost of capital is given by:

- ◯ $MPK + R + \bar{d}$
- ◯ $\frac{R\bar{d}}{MPK}$
- √ $R + \bar{d} - \frac{\Delta p_k}{p_k}$
- ◯ $R + \bar{d} - MPK$
- ◯ $\bar{d} + \Delta p_s$

(n) (1 point) Let P denote the price of goods in the United States, P^w denote the price of goods in the foreign country, and E the exchange rate, measured as the number of units of foreign currency that can be purchased with one dollar. According to the law of one price:

- ◯ $P = EP^w$
- √ $P^w = EP$
- ◯ $E = PP^w$
- ◯ $P^w = E + P$
- ◯ $P = E + P^w$

(o) (1 point) An explanation for a potential depreciation of the dollar vis-à-vis the euro is that:

- ◯ incomes are higher in the euro area than in the United States
- ◯ interest rates are higher in the United States than in the euro area
- √ **inflation in the United States is higher than in the euro area**
- ◯ there is less risk in the United States than in the euro area
- ◯ None of these answers are correct.

(p) (1 point) Which of the following can be used to explain the failure of the law of one price with respect to Big Macs?

- √ **transportation costs**
- ○ wage equalization across borders
- ○ differences in the money supply
- ○ over- or undervalued currency
- ○ real interest rate differences.

(q) (1 point) Free flow of international assets is desirable because it allows countries to:

- √ **borrow when times are bad in order to smooth consumption**
- ○ borrow when times are good in order to increase consumption
- ○ buy in higher value asset markets
- ○ hold gold
- ○ maintain price stability.

(r) (1 point) From the national income identity, we have:

- ○ $NX < 0 \quad \Rightarrow \quad (C - T) + I + G < Y.$
- ○ $NX > 0 \quad \Rightarrow \quad C + I + G > Y.$
- √ $\boldsymbol{NX < 0 \quad \Rightarrow \quad C + I + G > Y.}$
- ○ $NX < 0 \quad \Rightarrow \quad (C - T) + I + G > Y$
- ○ None of these answers is correct.

(s) (1 point) Suppose, in the North, one unit of labor produces f_n units of fish or c_n units of chips. In the South, one unit of labor produces f_s units of fish or c_s units of chips. For the South to specialize in fish, which of the following must be true?

- ○ $f_s > c_s$
- ○ $\frac{f_s}{c_s} > \frac{f_n}{c_n}$
- √ $\boldsymbol{\frac{c_s}{f_s} < \frac{c_n}{f_n}}$
- ○ $\frac{f_s}{c_s} < \frac{f_n}{c_n}$
- ○ $f_s c_s < f_n c_n.$

(t) (1 point) Free labor migration is more effective at improving welfare in low-income countries than moving capital because:

- √ **it moves labor to the place where productivity is high.**
- ○ low-income countries do not have much capital per person.
- ○ it moves labor to the place where there is a lot of capital per person.
- ○ it moves labor to the place where there is not much capital per person.
- ○ it is cheaper to move labor than capital.

2 Short-Run: an Oil Price Shock (12 points)

2. Take the usual AS/AD model, ruling out Aggregate Demand shocks, so with $\bar{a} = 0$, but assuming a one-time, unexpected oil price shock $\bar{o}_0 > 0$. One time means that the oil price shock lasts only for one period, in period $t = 0$, and that $\bar{o}_t = 0$ for all subsequent $t \in \{1, 2, ...\}$. Unexpected means that the economy was originally in steady-state, and in particular that $\pi_0 = \bar{\pi}$. Unless otherwise noted, agents have adaptive expectations about inflation. The economy is described by an AS/AD model. In particular, the AS curve is given by (be careful about the convention on the timing of the oil shock $\bar{o}_{t-1}$!):

$$\pi_t = \pi_t^e + \bar{\nu}\tilde{Y}_t + \bar{o}_{t-1}.$$

The AD curve is the standard one used throughout the course.

(a) (1 point) What are the values of π_1 and $\tilde{Y}_1$ in terms of the parameters of the model? (in particular the size of the oil price shock, $\bar{o}_0$)

Solution: The AS/AD equations are:

$$\pi_1 = \bar{\pi} + \bar{\nu}\tilde{Y}_1 + \bar{o}_0$$
$$\tilde{Y}_1 = -\bar{b}\bar{m}\left(\pi_1 - \bar{\pi}\right).$$

Using the second equation to plug in the first, one gets:

$$\tilde{Y}_1 = -\bar{b}\bar{m}\left[\bar{\nu}\tilde{Y}_1 + \bar{o}_0\right] \quad \Rightarrow \quad \tilde{Y}_1 = -\frac{\bar{b}\bar{m}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{o}_0.$$

Using this to replace in one or the other equation, one gets:

$$\pi_1 = \bar{\pi} + \frac{\bar{o}_0}{1 + \bar{b}\bar{m}\bar{\nu}}.$$

(b) (1 point) Show analytically[1] the effect of a more agressive monetary policy on inflation and short-run output in period 1: do inflation and short-run output increase or decrease with a more agressive monetary policy?

Solution:

$$\frac{\partial \pi_1}{\partial \bar{m}} = -\frac{\bar{b}\bar{\nu}}{(1 + \bar{b}\bar{m}\bar{\nu})^2}\bar{o}_0.$$
$$\frac{\partial \tilde{Y}_1}{\partial \bar{m}} = -\frac{\bar{b}\bar{\nu}}{(1 + \bar{b}\bar{m}\bar{\nu})^2}\bar{o}_0.$$

Both decrease. Since inflation was increasing initially, this means there is a more muted response of inflation. To the contrary, since output was already

[1] That is, in mathematical terms.

decreasing, this means that the response of short-run output is actually more important. With a more agressive monetary policy, the bulk of the adjustment goes through unemployment, and a decrease in short-run output. (Note: of course, adaptive expectations neglect the fact that if monetary policy was expected to be agressive, then people may anticipate that inflation will be lower in future periods, which on the contrary mitigates the needed adjustment of short-run output).

(c) (1 point) Illustrate this on two graphs with the AS/AD curves: show one AS/AD diagram with a soft monetary policy, and next to it another AS/AD diagram with an agressive monetary policy. Show π_1 and $\tilde{Y}_1$ as well as the long run values of inflation and short-run output on these graphs.

Solution: See TA section.

(d) (1 point) Eliminating $\tilde{Y}_t$ from the AS/AD model, find a difference equation[2] for π_t, for $t \in \{2, 3, ...\}$.

Solution: Because people have adaptive expectations, he have the following equations:

$$\pi_t = \pi_{t-1} + \bar{\nu}\tilde{Y}_t + \bar{o}_{t-1}$$
$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

Solving this system of two equations and two unknowns (where the unknowns are $(\tilde{Y}_t, \pi_t)$), it is easy to show that inflation follows the following difference equation:

$$\pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi} + \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\bar{o}_{t-1}.$$

Moreover we have that $\bar{o}_{t-1} = 0$ for all $t \in \{2, 3, ...\}$, because the shock is a one time shock. So finally:

$$\forall t \in \{2, 3, ...\}, \quad \pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi}$$

(e) (1 point) Substracting $\bar{\pi}$ on both sides in the difference equation for π_t, show that $\pi_t - \bar{\pi}$ satisfies a simpler difference equation than π_t.[3] Solve for this difference

[2] A difference equation is an expression of an economic quantity as a function of its previous (lagged) values, generally the value in the previous period. For example π_t expressed as a function of π_{t-1} is a difference equation for π_t

[3] Simpler in the sense that you can solve for it. For example, a simple difference equation is one of the form $u_t = \rho u_{t-1}$, whose solution is $u_t = \rho^{t-1}u_1$.

equation. This should give you an expression for π_t as a function of π_1. Then replace π_1 with the value found in question (a), to get π_t as a function of time and the parameters of the model.

Solution: We have:

$$\pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi} \quad \Rightarrow \quad \pi_t - \bar{\pi} = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\left(\pi_{t-1} - \bar{\pi}\right).$$

This difference equation iterates (just as those in the Romer model) through:

$$\pi_t - \bar{\pi} = \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t-1}(\pi_1 - \bar{\pi}).$$

Using the expression for π_1 in the previous question:

$$\pi_1 = \bar{\pi} + \frac{\bar{o}_0}{1+\bar{b}\bar{m}\bar{\nu}},$$

This gives:

$$\pi_t = \bar{\pi} + \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t}\bar{o}_0.$$

(f) (1 point) Use the (AD) curve to then calculate $\tilde{Y}_t$ as a function of time and the parameters of the model.

Solution: We use the AD curve which is:

$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

with the espression found in question (f):

$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t}\bar{o}_0.$$

(g) (1 point) Numerical Application: Suppose the parameters of the AS and AD curves take the following values: $\bar{o}_0 = 2\%$, $\bar{a} = 0$, $\bar{b} = 1/2$, $\bar{m} = 1/2$, $\bar{\nu} = 1/2$, and $\bar{\pi} = 2\%$. Solve for the value of short-run output and the inflation rate for the first 2 years after the shock. (express your result as a single fraction, since you do not have a calculator !)

Solution: Therefore, numerically:

$$\frac{1}{1+\bar{b}\bar{m}\bar{\nu}} = 8/9.$$

Time	Inflation π_t	Short-Run Output $\tilde{Y}_t$
0	2 %	0 %
1	$2+ 8/9 * 2 = 34/9\%$	$-4/9\%$
2	$2+ ((8/9)^2) * 2 = 290/81\%$	$-32/81\%$

(h) (1 point) Calculate how much realized inflation differs from expected inflation, or $\pi_t - \pi_t^e$, for any $t \geq 1$, in this model. Simplify the expression so that its sign appears clearly.[4]

Solution: We have that:

$$\pi_t - \pi_t^e = \pi_t - \pi_{t-1} = \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^t \bar{o}_0 - \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t-1} \bar{o}_0$$

$$\pi_t - \pi_t^e = -\frac{\bar{b}\bar{m}\bar{\nu}}{(1+\bar{b}\bar{m}\bar{\nu})^{t-1}}\bar{o}_0 < 0$$

(i) (1 point) Represent on a graph the behavior of inflation π_t as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\bar{\pi}$ and π_1 on this graph.

Solution: You should have drawn a graph with inflation on the y axis, and time on the x axis. Inflation first shoots up to π_1 and then comes back to steady state $\bar{\pi}$.

(j) (1 point) Represent on a graph the behavior of short-run output $\tilde{Y}_t$ as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\tilde{Y}_\infty$ and $\tilde{Y}_1$ on this graph.

Solution: You should have drawn a graph with short-run output on the y axis, and time on the x axis. Short-run output becomes negative at $t = 1$ and then converges back to steady state (0).

(k) (1 point) Why are these adaptive expectations where $\pi_t^e = \pi_{t-1}$ considered as "non rational" ? What is irrational about them?

Solution: Agents consistently overestimate what inflation will be next period. They don't understand that inflation tomorrow will be determined by the reaction of the central bank, which will tighten monetary policy in the face of too high inflation and therefore lower inflation. That is irrational. Moreover, this is a mistake that agents could learn not to make over time, as they are always making the same mistake, in the same direction. But under adaptive expectations, they do not.

[4] That is, it should be clear from your expression of $\pi_t - \pi_t^e$ whether it is positive or negative

(l) (1 point) What would $\pi_t - \pi_t^e$ be equal to under rational expectations?

Solution: Under rational expectations, we would have:

$$\pi_t - \pi_t^e = 0.$$

3 Short-Run: an Aggregate Demand Shock (18 points)

3. Take the usual AS/AD model, ruling out Oil Price shocks, so with $\bar{o} = 0$, but assuming a one-time, unexpected aggregate demand shock $\bar{a}_1 > 0$, materializing in the period 1 (AD) demand equation. One time means that the aggregate demand shock lasts only for one period, in period $t = 1$, and that $\bar{a}_t = 0$ for all subsequent $t \in \{2, 3, ...\}$, and also that $\bar{a}_0 = 0$. Unexpected means that the economy was originally in steady-state, so that $\tilde{Y}_0 = 0$ and $\pi_0 = \bar{\pi}$. Unless otherwise noted, agents have adaptive expectations about inflation. The economy is described by an AS/AD model. In particular, the AD curve is given by (be careful about the convention on the timing of the aggregate demand shock $\bar{a}_t$!):

$$\tilde{Y}_t = \bar{a}_t - \bar{b}\bar{m}(\pi_t - \bar{\pi}).$$

From the above assumptions on the aggregate demand shock, the AD curve is given as follows for $t \in \{0, 2, 3, ...\}$:

$$\tilde{Y}_t = -\bar{b}\bar{m}(\pi_t - \bar{\pi}).$$

For $t = 1$, the AD curve is in contrast given by:

$$\tilde{Y}_1 = \bar{a}_1 - \bar{b}\bar{m}(\pi_1 - \bar{\pi}).$$

The AS curve is the standard one used throughout the course, and has no oil price shock.

(a) (2 points) What are the values of π_1 and $\tilde{Y}_1$ in terms of the parameters of the model? (in particular the size of the aggregate demand shock, $\bar{a}_1$)

Solution: The AS/AD equations for period 1 are, because $\pi_0 = \bar{\pi}$:

$$\begin{aligned} \pi_1 &= \bar{\pi} + \bar{\nu}\tilde{Y}_1 \\ \tilde{Y}_1 &= \bar{a}_1 - \bar{b}\bar{m}\left(\pi_1 - \bar{\pi}\right). \end{aligned}$$

Using the second equation to plug in the first, one gets:

$$\pi_1 = \bar{\pi} + \bar{\nu}\left[\bar{a}_1 - \bar{b}\bar{m}(\pi_1 - \bar{\pi})\right] \quad \Rightarrow \quad \pi_1 = \bar{\pi} + \frac{\bar{\nu}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{a}_1.$$

Using this to replace in one or the other equation, one gets:

$$\tilde{Y}_1 = \frac{1}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{a}_1.$$

Finally the solution is:

$$\pi_1 = \bar{\pi} + \frac{\bar{\nu}}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{a}_1 \qquad \tilde{Y}_1 = \frac{1}{1 + \bar{b}\bar{m}\bar{\nu}}\bar{a}_1.$$

(b) (2 points) Show analytically[5] the effect of a more agressive monetary policy on inflation and short-run output in period 1: do inflation and short-run output increase or decrease with a more agressive monetary policy?

Solution:

$$\frac{\partial \pi_1}{\partial \bar{m}} = -\frac{\bar{b}\bar{\nu}^2}{(1+\bar{b}\bar{m}\bar{\nu})^2}\bar{a}_1.$$
$$\frac{\partial \tilde{Y}_1}{\partial \bar{m}} = -\frac{\bar{b}\bar{\nu}}{(1+\bar{b}\bar{m}\bar{\nu})^2}\bar{a}_1.$$

Both decrease. Since inflation was increasing initially, this means there is a more muted response of inflation. Similarly, there is a more muted response of short-run output, as monetary policy dampens the effects of the aggregate demand shock.

(c) (1 point) Illustrate this on two graphs with the AS/AD curves: show one AS/AD diagram with a soft monetary policy, and next to it another AS/AD diagram with an agressive monetary policy. Show π_1 and $\tilde{Y}_1$ as well as the long run values of inflation and short-run output on these graphs.

Solution: See TA section.

(d) (1 point) Eliminating $\tilde{Y}_t$ from the AS/AD model, find a difference equation[6] for π_t, for $t \in \{2, 3, ...\}$.

Solution: Because people have adaptive expectations, he have the following equations:

$$\pi_t = \pi_{t-1} + \bar{\nu}\tilde{Y}_t$$
$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

Solving this system of two equations and two unknowns (where the unknowns are $(\tilde{Y}_t, \pi_t)$), it is easy to show that inflation follows the following difference equation:

$$\forall t \in \{2, 3, ...\}, \quad \pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi}$$

(e) (2 points) Substracting $\bar{\pi}$ on both sides in the difference equation for π_t, show that

[5] That is, in mathematical terms.

[6] A difference equation is an expression of an economic quantity as a function of its previous (lagged) values, generally the value in the previous period. For example π_t expressed as a function of π_{t-1} is a difference equation for π_t

$\pi_t - \bar{\pi}$ satisfies a simpler difference equation than π_t.[7] Solve for this difference equation. This should give you an expression for π_t as a function of π_1. Then replace π_1 with the value found in question (a), to get π_t as a function of time and the parameters of the model.

Solution: We have:

$$\pi_t = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\pi_{t-1} + \frac{\bar{b}\bar{m}\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{\pi} \quad \Rightarrow \quad \pi_t - \bar{\pi} = \frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\left(\pi_{t-1} - \bar{\pi}\right).$$

This difference equation iterates (just as those in the Romer model) through:

$$\pi_t - \bar{\pi} = \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t-1}(\pi_1 - \bar{\pi}).$$

Using the expression for π_1 in the previous question:

$$\pi_1 = \bar{\pi} + \frac{\bar{\nu}}{1+\bar{b}\bar{m}\bar{\nu}}\bar{a}_1.$$

This gives:

$$\pi_t = \bar{\pi} + \left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t}\bar{\nu}\bar{a}_1.$$

(f) (2 points) Use the (AD) curve to then calculate $\tilde{Y}_t$ as a function of time and the parameters of the model, for $t \in \{2, 3, ...\}$.

Solution: We use the AD curve which is:

$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\pi_t - \bar{\pi}\right).$$

with the espression found in question (f):

$$\tilde{Y}_t = -\bar{b}\bar{m}\left(\frac{1}{1+\bar{b}\bar{m}\bar{\nu}}\right)^{t}\bar{\nu}\bar{a}_1.$$

(g) (1 point) What is the sign of $\tilde{Y}_t$ (positive or negative?) for $t \in \{2, 3, ...\}$?

Solution: The sign of $\tilde{Y}_t$ is negative for $t \in \{2, 3, ...\}$.

(h) (3 points) What is the economic intuition for that ? Explain precisely.

[7] Simpler in the sense that you can solve for it. For example, a simple difference equation is one of the form $u_t = \rho u_{t-1}$, whose solution is $u_t = \rho^{t-1}u_1$.

Solution: The reason is that the aggregate demand shock has disappeared in period 2, period 3, etc. but inflation is now higher than it used to be. Because of adaptive expectations, the central bank needs to respond to this inflation by tolerating negative short-run output for a while.

(i) (1 point) Represent on a graph the behavior of inflation π_t as a function of time. In particular, represent $t = 0$, $t = 1$, $t = +\infty$, $\bar{\pi}$ and π_1 on this graph.

Solution: You should have drawn a graph with inflation on the y axis, and time on the x axis. Inflation first shoots up to π_1 and then comes back to steady state $\bar{\pi}$.

(j) (2 points) Represent on a graph the behavior of short-run output $\tilde{Y}_t$ as a function of time. In particular, represent $t = 0$, $t = 1$, $t = 2$, $t = +\infty$, and $\tilde{Y}_0$, $\tilde{Y}_1$, $\tilde{Y}_2$, $\tilde{Y}_\infty$ on this graph.

Solution: You should have drawn a graph with short-run output on the y axis, and time on the x axis. **Short-run output first becomes positive for** $t = 1$, and **then becomes negative** for $t = 2$, then goes back to 0, always staying in the negative territory.

(k) (1 point) Why is this exercice illustrating the fundamental reason why central banks want to stabilize short-run output?

Solution: Aggregate demand shocks are good in the short-run, as they boost short-run output. However, they also lead to increased inflation, which must be offset by monetary policy through negative short-run output. As Jones puts it page 357: "booms are matched with recessions. In some "average" sense, the economy does not really gain in terms of output: the boom and recession offset each other." However, the costs of inflation are real, and so the central bank would prefer to avoid these loops altogether, which are neutral on average in terms of output, but also lead to higher than targeted inflation.

4 Long-Run: The Solow Growth model (7 points)

4. Consider an Solow economy with the following Cobb-Douglas, constant returns to scale, production function:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}.$$

Assume that of this production, a fraction $\bar{s}$ is saved (and invested), and a fraction $1 - \bar{s}$ is consumed. Assume that the labor force is fixed: $L_t = \bar{L}$. Assume that capital depreciates at rate $\bar{d}$.

(a) (1 point) Write the law of motion for capital.[8] Solve for the steady state value of capital K^*, and the steady state value of Y^*, in this economy.

Solution:

$$\begin{aligned}\Delta K_{t+1} = K_{t+1} - K_t &= \bar{s} Y_t - \bar{d} K_t \\ &= \bar{s} \bar{A} K_t^{1/3} \bar{L}^{2/3} - \bar{d} K_t\end{aligned}$$

Steady-state obtains with $\Delta K_{t+1} = 0$, so that:

$$\bar{s} Y^* = \bar{d} K^*.$$

The production function gives a second equation:

$$\bar{s} Y^* = \bar{A} \left(K^*\right)^{1/3} \bar{L}^{2/3}.$$

Again, this is a two equations/two variables system whose solution is:

$$K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{3/2} \bar{L} \qquad Y^* = \left(\frac{\bar{s}}{\bar{d}}\right)^{1/2} \left(\bar{A}\right)^{3/2}.$$

(b) (1 point) Comment the dependence of Y^* with $\bar{A}$. Compare with the production model. Is there a difference? Why?

Solution: There is an exponent 3/2. The production function has linearity in total factor productivity. The reason is that capital accumulation amplifies productivity differences, because more productive countries accumulate more capital, which increases their production further.

(c) (1 point) Draw the Solow Diagram in this case: represent capital K on the x axis, and investment, depreciation, and output, on the y axis. Represent Y^* and K^* on this diagram.

[8] A law of motion for u_t is an expression of the form: u_{t+1} as a function of u_t, or Δu_{t+1} as a function of u_t.

Solution: See TA section.

(d) (1 point) Draw the same diagram again, but only with depreciation and investment on the y axis. Show on the graph how the economy converges to the steady state if it starts with a higher than steady-state level of capital.

Solution: See TA section.

(e) (1 point) Imagine that the government can enact policies aimed at targeting people's savings rate $\bar{s}$. Which savings rate would maximize steady-state production ? What would steady-state consumption then be equal to?

Solution: Y^* is maximized when savings are maximum, so the savings rate maximizing steady state production would be $\bar{s} = 1$. However people would be starving in this economy. All its output would be used towards increased capital accumulation.

(f) (1 point) Using that the steady state value for consumption is equal to $C^* = (1 - \bar{s})Y^*$, show that the savings rate $\bar{s}$ has two opposing effects on consumption. Give an intuition.

Solution: $(1 - \bar{s})$ decreases in $\bar{s}$ while Y^* increases in $\bar{s}$. On the one hand, steady state consumption increases when savings increase because there is then more production. But on the other, consumption decreases because a higher fraction of production is then saved rather than consumed.

(g) (1 point) Adam Smith once wrote: "Consumption is the sole end and purpose of all production". Calculate the steady-state consumption maximizing level of the savings rate $\bar{s}$ in the Solow model.

Solution: You need to maximize the following:

$$Y^* = (1 - \bar{s}) \left(\frac{\bar{s}}{\bar{d}}\right)^{1/2} \left(\bar{A}\right)^{3/2}.$$

The first order condition gives (note you actually just need to maximize $(1 - \bar{s})(\bar{s})^{1/3}$):

$$\frac{\partial Y^*}{\partial \bar{s}} = 0 \quad \Rightarrow \quad -\bar{s}^{1/2} + \frac{1}{2}\bar{s}^{-1/2}(1 - \bar{s}) = 0 \quad \Rightarrow \quad \boxed{\bar{s} = \frac{1}{3} = 33.3\%}.$$

5 Long-Run: the AK model (8 points)

5. Consider a Solow model where the production function no longer exhibits diminishing returns to capital accumulation. In other words, now $Y_t = \bar{A}K_t$. This is not particularly realistic, for reasons discussed during the course. But it is interesting to consider this case nonetheless because of what it tells us about the workings of the Solow model. The rest of the Solow model is unchanged: the savings rate is still $\bar{s}$, and the depreciation rate is still $\bar{d}$. The initial capital stock is given, equal to $\bar{K}_0$.

   (a) (1 point) Write the law of motion for capital.

   > **Solution:** The law of motion for capital is:
   >
   > $$\Delta K_{t+1} = \bar{s}\bar{A}K_t - \bar{d}K_t.$$

   (b) (2 points) Solve for K_t analytically[9] as a function of K_0, time t, and the parameters of the model. Solve for Y_t analytically, also as a function of K_0, time t, and the parameters of the model.

   > **Solution:** Solving analytically for K_t yields:
   >
   > $$K_{t+1} = \left(1 + \bar{s}\bar{A} - \bar{d}\right) K_t \quad \Rightarrow \quad K_t = \left(1 + \bar{s}\bar{A} - \bar{d}\right)^t K_0.$$
   >
   > Solving analytically for Y_t yields:
   >
   > $$Y_t = \bar{A}K_t = \bar{A}\left(1 + \bar{s}\bar{A} - \bar{d}\right)^t K_0.$$

   (c) (1 point) Assume that the savings rate is relatively high, and in particular that $\bar{s} > \frac{\bar{d}}{\bar{A}}$. What happens to the capital stock and to output over time? Give an economic intuition for this result.

   > **Solution:** Both increase to infinity over time because: $1 + \bar{s}\bar{A} - \bar{d} > 1$.
   > The economic intuition is that depreciation is lower than savings.

   (d) (1 point) Illustrate this with the help of a Solow Diagram. Show K_0 on this graph, K_1, K_2, etc.

   > **Solution:** See TA Section

   (e) (1 point) Assume that the savings rate is relatively low, in the sense that $\bar{s} < \frac{\bar{d}}{\bar{A}}$. What happens to the capital stock and to output over time? What is the economic intuition for this result?

[9] Analytically means in mathematical terms.

Solution: They both decrease to 0 over time because: $1 + \bar{s}\bar{A} - \bar{d} < 1$.
The economic intuition is that depreciation is higher than savings.

(f) (1 point) Illustrate this with the help of a Solow Diagram. Show K_0 on this graph, K_1, K_2, etc.

Solution: See TA Section

(g) (1 point) Assume that the savings rate is exactly such that $\bar{s} = \frac{\bar{d}}{\bar{A}}$. What happens then? Why?

Solution: We then have : $K_{t+1} = K_t$ and $Y_t = \bar{A}K_0$.

6 Microfoundations: The Neoclassical Consumption Model with Log Utility and $\beta \neq 1$ (15 points)

6. Consider a consumer consuming goods in period 0 ("today") in quantity c_0, and goods in period 1 ("tomorrow") in quantity c_1. Assume that in period 0, the consumer is born with a financial wealth equal to f_0, and that he chooses his level of **savings** f_1 (or, equivalently, of **saving** $f_1 - f_0$) to maximize a lifetime utility function given by:

$$U = \log(c_0) + \beta \log(c_1).$$

The consumer also earns a labor income equal to y_0 in period 0, and equal to y_1 in period 1. The interest rate he earns on his savings is given by R.

(a) (1 point) Write the two budget constraints that the consumer faces in period 0, and period 1.

Solution: The budget constraint in period 0 is:

$$c_0 = y_0 - (f_1 - f_0).$$

The budget constraint that the consumer faces in period 1 is:

$$c_1 = y_1 + (1 + R)f_1.$$

(b) (2 points) Eliminate f_1 from these equations so as to write the consumer's intertemporal budget constraint. Show the steps you use to get to this expression. (you won't get any credit if you don't)

Solution: Express f_1 from budget constraint in period 1 and get:

$$f_1 = \frac{c_1 - y_1}{1 + R}.$$

Then replacing in the first gives:

$$c_0 = y_0 - \left(\frac{c_1 - y_1}{1 + R} - f_0\right).$$

Therefore the intertemporal budget constraint is:

$$c_0 + \frac{c_1}{1 + R} = f_0 + y_0 + \frac{y_1}{1 + R}.$$

(c) (2 points) Maximize the lifetime utility function of the consumer under the previous intertemporal budget constraint to derive the Euler equation. (again, please give details to get credit)

Solution: See Assignment 6.

(d) (1 point) What is the intuition for this equation?

Solution: The marginal utility from consuming in period 1 is $\beta u'(c_1) = \beta/c_1$. The marginal utility from consuming in period 0 is $u'(c_0) = 1/c_0$. By putting one unit of consumption in the bank, one forgoes 1 unit of consumption in period 0 to get $1 + R$ units of consumption in period 1. The two have to be equal:

$$\frac{1}{c_0} = (1+R)\beta\frac{1}{c_1}.$$

(e) (1 point) Using the Euler equation and the intertemporal budget constraint, solve for c_0 and c_1 as a function of the parameters of the model.

Solution: The two equations are:

$$\frac{c_1}{c_0} = \beta(1+R)$$
$$c_0 + \frac{c_1}{1+R} = f_0 + y_0 + \frac{y_1}{1+R}.$$

Therefore:

$$c_1 = \beta(1+R)c_0 \quad \Rightarrow \quad c_0 + \beta c_0 = f_0 + y_0 + \frac{y_1}{1+R}$$
$$\Rightarrow \quad c_0 = \frac{1}{1+\beta}\left(f_0 + y_0 + \frac{y_1}{1+R}\right).$$

Now you can calculate consumption at time 1, for example from the first equation:

$$c_1 = (1+R)\frac{\beta}{1+\beta}\left(f_0 + y_0 + \frac{y_1}{1+R}\right).$$

(f) (1 point) What is the marginal propensity to consume out of current earnings?

Solution: As the previous expression shows, the marginal propensity to consume out of current earnings is $1/(1+\beta)$.

(g) (1 point) Why does it not depend on the interest rate?

Solution: The logarithmic utility has canceling income and substitution effects. On the one hand, income effects lead the consumer to consume more because he is richer. On the other, a higher interest rate is attractive for saving which leads the consumer to consume less. The two effects exactly offset each other.

(h) (1 point) How does it vary with β ? What is the intuition for that?

Solution: It decreases with β. The reason is that with less impatience, the consumer consumes less of an increase in current income, and saves more of this increase.

(i) (1 point) What is the marginal propensity to consume out of future earnings?

Solution: It is given by $1/[(1+\beta)(1+R)]$.

(j) (1 point) Why does it depend on the interest rate?

Solution: Because future income is given later, its value today is a function of the interest rate. It is still true that income and substitution effects cancel out, but the increase in income is not 1 for one but attenuated by a factor $1/(1+R)$. Therefore the substitution effect dominates, and an increase in the interest rate leads to a decrease in consumption.

(k) (1 point) What is the marginal propensity to consume out of current wealth f_0?

Solution: Same: $1/(1+\beta)$.

(l) (2 points) Assume that $\beta = 1$, and that a consumer were to live and consume 70 periods instead of just 2. How would you then write his lifetime utility function? What would marginal propensity to consume out of wealth then be?

Solution: The new lifetime utility function would be [be careful: **it must go until 69** !!]:

$$U = \log(c_0) + \log(c_1) + ... + \log(c_{69}).$$

The marginal propensity to consume would then be 1/70.

7 Long-Run / Microfoundations: a Growth model with an endogenous saving behavior (20 points + 3 bonus points)

7. Consider a closed economy with the following Cobb-Douglas, constant returns to scale, production function:

$$Y_t = \bar{A}K_t^{1/3}L_t^{2/3}.$$

Assume that the labor force is fixed to unity: $L_t = \bar{L} = 1$. Assume that labor is taxed at rate τ (that is, if the employer pays w_t in wages, the worker receives only $(1-\tau)w_t$). Assume that capital depreciates at rate $\bar{d} = 1 = 100\%$. (that is, capital fully depreciates each period)[10]

(a) (1 point) Assuming firms are competitive and maximize profits, what is the wage w_t paid by employers in period t as a function of $\bar{A}$ and the level of the capital stock in period t, K_t ?

Solution: The wage paid by employers is given by:

$$w_t = \frac{2}{3}\bar{A}K_t^{1/3}\bar{L}^{-1/3}$$

Therefore, because $\bar{L} = 1$, we have:

$$w_t = \frac{2}{3}\bar{A}K_t^{1/3}.$$

(b) (1 point) What is the wage received by workers, as a function of τ, $\bar{A}$ and the level of the capital stock in period t, K_t ?

Solution: The net of tax wage is given by:

$$(1-\tau)w_t = (1-\tau)\frac{2}{3}\bar{A}K_t^{1/3}.$$

(c) (2 points) Represent the labor market (labor supply and labor demand), with employment on the x axis, and the wage w_t on the y axis. Show the effect of the imposition of a tax. Show the deadweight loss.

Solution: See TA section

(d) (1 point) Who effectively pays the tax? What is the economic intuition for that?

[10] We will later interpret the length of time between t and $t+1$ as a generation, or about $30 - 40$ years.

Solution: The worker pays the tax. This is because we assumed that the labor force was fixed.

(e) (1 point) Taking K_t as given, what is the effect of the tax on output Y_t?

Solution: There is no effect of the tax on output, as given K_t, output is given by:

$$Y_t = \bar{A} K_t^{1/3},$$

where τ does not appear.

From now on, and until the end of this exercice, we assume that $\tau = 0$. Moreover, instead of assuming that savings are a fixed fraction $\bar{s}$ of output, we will instead derive the saving behavior from microfoundations.

We assume that people in this economy live only for two periods. However, instead of calling them "0" and "1", or "today" and "tomorrow", we now call them "t" and "$t+1$". People are called "young" in the first period of their life, and "old" in the second. People from generation t are young in period t, and old in period $t+1$. We denote their consumption when young by c_t^y and their consumption when old by c_{t+1}^o. In terms of the lectures, you should really think of c_t^y as c_0, and of c_{t+1}^o as c_1. People work when young, and then receive a wage given by w_t (remember, we assumed $\tau = 0$). They retire when old, and then do not work. Their lifetime utility is logarithmic with $\beta = 1$:

$$U = \log(c_t^y) + \log(c_{t+1}^o).$$

Their intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+R} = w_t.$$

There is always two generations living in period t: the previous period's young, born in period $t-1$, now old, consuming the return from their savings; and this period's young, newly born (in period t).

(f) (2 points) Calculate c_t^y and c_{t+1}^o as a function of w_t. (you can give the solution without an explanation) Calculate saving by the young, given by $w_t - c_t^y$.

Solution: We have:

$$c_t^y = \frac{w_t}{2} \qquad c_{t+1}^o = (1+R)\frac{w_t}{2}.$$

Savings is given by:

$$w_t - c_t^y = \frac{w_t}{2}.$$

(g) (1 point) What is the relationship between savings by the young and investment in this economy? (Hint: remember that the economy is closed !)

Solution: Because it is a closed economy, we have that net exports are zero, and savings must finance investment at home:

$$I_t = s_t = w_t - c_t^y = \frac{w_t}{2}.$$

(h) (3 points) Write the law of motion for capital as a function of the parameters of the model. Note that it takes a very similar form as that in the Solow model. What is the savings rate in this model corresponding to $\bar{s}$ in the Solow model?

Solution: We have that (since $\bar{d} = 1$):

$$\Delta K_{t+1} = I_t - K_t$$

Therefore:

$$\Delta K_{t+1} = \frac{w_t}{2} - K_t.$$

Because we have:

$$w_t = \frac{2}{3} Y_t.$$

The law of motion for capital is therefore:

$$\Delta K_{t+1} = \frac{1}{3} \bar{A} K_t^{1/3} - \bar{d} K_t.$$

This is the capital accumulation equation (or law of motion for capital) of the Solow model, with $\bar{s} = 1/3$.

(i) (1 point) Write the arbitrage condition for the firm between the marginal product of capital, depreciation, and the interest rate R, assuming that the price of capital is constant and equal to one.

Solution: The arbitrage condition is: $R = MPK - \bar{d} = MPK - 1$.

(j) (2 points) What is an expression for R as a function of K_t and the parameters of the model?

Solution: We have that:

$$MPK = \frac{1}{3}\bar{A}K_t^{-2/3}.$$

Therefore:

$$R = \frac{1}{3}\bar{A}K_t^{-2/3} - \bar{d} = \frac{1}{3}\bar{A}K_t^{-2/3} - 1.$$

Assume now that people work half time when young, and half time when old, earning wage $w_t/2$ in each period (total labor supply is still 1 in each period), so that the new intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+R} = \frac{w_t}{2} + \frac{w_t}{2(1+R)}.$$

(k) (3 points) Write the law of motion for capital ONLY as a function of the parameters of the model. (in particular, substitute out R)

Solution: Savings are now given by:

$$\frac{w_t}{2} - \frac{1}{2}\left(\frac{w_t}{2} + \frac{w_t}{2(1+R)}\right) = \frac{w_t}{4} - \frac{w_t}{4(1+R)} = \frac{w_t}{4}\frac{R}{1+R}.$$

Therefore the law of motion for capital is:

$$\Delta K_{t+1} = \frac{w_t}{4}\frac{R}{1+R} - \bar{d}K_t.$$

Simpler is to use this non simplified expression:

$$\Delta K_{t+1} = \frac{w_t}{4}\left(1 - \frac{1}{1+R}\right) - \bar{d}K_t.$$

One uses:

$$1 + R = \frac{1}{3}\bar{A}K_t^{-2/3} \qquad w_t = \frac{2}{3}\bar{A}K_t^{1/3}.$$

Therefore, the law of motion for capital is, with $\bar{d} = 1$:

$$\Delta K_{t+1} = \frac{1}{6}\bar{A}K_t^{1/3} - \frac{1}{2}K_t - \bar{d}K_t.$$

Therefore:

$$\Delta K_{t+1} = \frac{1}{6}\bar{A}K_t^{1/3} - \frac{3}{2}K_t.$$

(l) (2 points) Why does it differ strikingly from the Solow model?

Solution: Note that in order to have the previously derived law of motion square with the Solow model, one would need the following parameters: $\bar{s} = 1/6$, and $\bar{d} = 150\%$. The implied depreciation rate, of course, does not make any economic sense: would be impossible that more than the previously accumulated capital stock depreciate each period.

The reason why this model differs strikingly from the Solow model is that because of the wealth effect, R impacts also how much is saved today. When the capital stock is higher, R is lower, and savings are also lower as a consequence. Therefore there are two things that enter negatively in the law of motion for capital:

- Depreciation: the usual $\bar{d}K_t$ term.
- The effect of interest rates (wealth effects): the (unusual) $\frac{1}{2}K_t$ term.

(m) (3 points) **Bonus:** With an analogy to the Solow model, explain how you could can nevertheless solve for this model. Show that the steady-state level of capital accumulation is quite low. What is the intuition for that? (use the back of the sheet if needed)

Solution: To get the dynamics, instead of plotting production and depreciation, plot $\frac{1}{6}\bar{A}K_t^{1/3}$ and $\frac{3}{2}K_t$. That is, the depreciation line is now replaced by the depreciation + wealth effects line.

The solution of the problem is the same as the solow model but with $\bar{s} = 1/6$ and $\bar{d} = 3/2$. The steady state level of capital accumulation is never so low in the Solow model, for economically we never have that $\bar{d}$ is ever higher than 100%.

However the steady state of the model is still given by $\Delta K_{t+1} = 0$ which yields:

$$\bar{s}\bar{A}\left(K^*\right)^{1/3} - \bar{d}K^* - \frac{1}{2}K^* = 0$$

That gives:

$$K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d} + 1/2}\right)^{3/2}.$$

The steady state level of the capital stock is quite low because of the added 1/2 term on the denominator, relative to Solow.