Macroeconomic Theory 102
Winter 2015 - François Geerolf
Midterm 1
Wednesday, January 28, 2015
Time Limit: 75 Minutes

Last Name: ____________________

First Name: ____________________

Teaching Assistant: ____________________

Student ID Number: __________

Signature ____________________

Test A

This exam contains 9 pages (including this cover page). You can earn 100 points + 6 bonus points. If you are stuck at some point, don't forget to answer the easy questions worth many points ! (there are many of them)

Instructions:

1. Print your Last name, First Name, Teaching Assistant Name (as a reminder, teaching assistants are: Flavien Moreau, Keyyong Park, Matias Vieyra, and Gabriel Zaourak), Student ID Number and Signature at the top of this page.
2. The only items which should be on your desk are pencils and/or pens. NO other items are allowed. Place any other item UNDER your desk. Calculators are NOT allowed.
3. Once the exam begins, you are not allowed to leave the room until you hand in your exam.

Good luck ! Budget your time wisely ! (skip the question or even the exercice if you get stuck)

Grade Table (FOR TEACHER USE ONLY)

Question	Points	Score
1	10	
2	14	
3	6	
4	20	
5	20	
6	36	
Total:	106	

Multiple Choice (10 points)

1. (10 points) These are **multiple choice** questions. Mark box if true.

   (a) (2 points) If per capita GDP in 2013 was \$20000, and per capita GDP in 2015 is \$20200, then how much approximately was the growth rate of per capita GDP every year?

   ◯ 1%
   ◯ 10%
   √ **0.5%**

   (b) (2 points) Over the past 50 years, Brazil's population growth rate has averaged about 2.3 percent. According to the rule of 70, Brazil's population will double in about:

   ◯ 3 years.
   √ **30 years.**
   ◯ 33 years.
   ◯ 161 years.
   ◯ 1.6 years.

   (c) (2 points) The key difference between the Solow model and the production model is:

   √ **The Solow model endogenizes the process of capital accumulation.**
   ◯ The standard model endogenizes the process of capital accumulation.
   ◯ The Solow model uses different values for the capital share.
   ◯ The Solow model does not contain a productivity measure.
   ◯ The Solow model exogenizes the process of capital accumulation.

   (d) (2 points) In the Solow model, if we assume that capital depreciation rates are the same across all countries, differences in per capita output can be explained by:

   ◯ the steady state capital stock.
   ◯ the initial capital stock and saving rates.
   √ **differences in productivity and saving rates.**
   ◯ the labor stock and saving rates.
   ◯ None of these answers are correct.

   (e) (2 points) An implication of the Solow model is that once an economy reaches the steady state:

   √ **per capita consumption is constant.**
   ◯ per capita output is constant, but per capita capital is not
   ◯ per capita capital is variable
   ◯ per capita consumption continues to grow
   ◯ per capita consumption is growing.

Two Questions directly from the course (20 points)

2. (14 points) Assume that production is given by the following production function: $Y_t = A_t K_t^a L_t^b$, and that $A_t = \bar{A}$. Give **short answers**, you won't be penalized if you do not write sentences. Go quickly to the next sections.

(a) (2 points) What is the name of this production function?

Solution: This is a Cobb-Douglas production function.

(b) (2 points) What is the relationship between a and b in this production function, if returns to scale are constant? (with respect to capital and labor)

Solution: $a + b = 1$

(c) (2 points) Why do macroeconomists use this production function, a power function of the quantity of capital and the quantity of labor? What statistical regularity makes them confident to use this function?

Solution: The fact that the share of labor in GDP, and the share of capital in GDP are constants. For example, the marginal product of capital shows that the share of capital is constant:

$$r_t = a\frac{Y_t}{K_t} \quad \Rightarrow \quad \frac{r_t K_t}{Y_t} = a.$$

Similarly, the marginal product of labor is given by:

$$w_t = b\frac{Y_t}{L_t} \quad \Rightarrow \quad \frac{w_t L_t}{Y_t} = b.$$

(d) (2 points) What are a and b equal to typically?

Solution: $a = 1/3$ and $b = 2/3$.

(e) (2 points) Why are they equal to these numbers?

Solution: The share of capital in value added is approximately 1/3, and that of labor is approximately 2/3.

(f) (2 points) Write an example of a production function with the same inputs (technology A_t, capital K_t and labor L_t), but decreasing returns to scale. (with respect to capital and labor)

Solution: There are many options. One is: $Y_t = A_t K_t^{1/4} L_t^{1/4}$.

(g) (2 points) Write an example of a production function with the same inputs, but increasing returns to scale. (again, with respect to capital and labor)

Solution: Again, many possibilities. One is: $Y_t = A_t K_t^{2/3} L_t^{2/3}$, but you could use your imagination.

3. (6 points) Quantity indexes.

(a) (3 points) Define the Laspeyres index and the Paasche index.

Solution: The Laspeyres index calculated changes in real GDP using the initial prices. The Paasche index, in contrast, calculates changes in real GDP using the final year prices.

(b) (3 points) What is the Fischer (or chain-weighted) index?

Solution: It is an average of the Laspeyres and the Paasche indexes.

Exercice 1 (20 points)

4. (20 points) Consider the following model of the labor market, with labor supply L^s and labor demand L^d respectively given as a function of the wage by:

$$L^s = \bar{a}w + \bar{l}$$
$$L^d = \bar{f} - w.$$

(a) (2 points) In the following box, represent this model graphically, as well as its equilibrium (L^*, w^*). Represent an increase in $\bar{a}$ on the graph, using a dotted line. Represent the effect of this increase in $\bar{a}$ on the wage and the effect on employment.

Solution: See the TA section.

(b) (2 points) Again, represent the labor market model in the following box, but this time consider a decrease in $\bar{f}$. Represent the effect on the wage and the effect on employment.

Solution: See the TA section.

(c) (4 points) One last time, represent the labor market model in the following box, but consider the introduction of a proportional tax τ on wages such that when w is paid by employers, employees actually only receive $w(1-\tau)$. Show the triangle corresponding to deadweight loss (also known as Harberger's (1924-) triangle).

> **Solution:** See the TA section.

(d) (4 points) Solve analytically for the equilibrium values of L^* and w^* in this model with no taxes.

> **Solution:** Using a little algebra, one can show that:
>
> $$w^* = \frac{\bar{f} - \bar{l}}{1 + \bar{a}} \qquad L^* = \frac{\bar{a}\bar{f} + \bar{l}}{1 + \bar{a}}$$

(e) (4 points) Show analytically the effect on the L^* and w^* of the two previous experiments (increase in $\bar{a}$, decrease in $\bar{f}$).

> **Solution:** When $\bar{a}$ increases, w decreases. You can see that through the derivative, or just because $\bar{a}$ is in the denominator and the wage has to be positive so $\bar{f} - \bar{l}$. The comparative statics with respect to $\bar{a}$ are slightly more complicated:
>
> $$\frac{\partial L^*}{\partial \bar{a}} = \frac{\bar{f} - \bar{l}}{(1 + \bar{a})^2}.$$
>
> Because the wage is positive in equilibrium, this derivative also is positive.

(f) (4 points) Show analytically the effect on the equilibrium wage w^* and employment L^* of the tax τ, as a function of τ.

> **Solution:** Labor demand is given by what the employers pay, wage w, but employees receive only $w(1-\tau)$, so that:
>
> $$L^s = \bar{a}w(1-\tau) + \bar{l}$$
> $$L^d = \bar{f} - w.$$
>
> Note that it is as if $\bar{a}$ had switched to $\bar{a}(1-\tau)$. Therefore:
>
> $$w^* = \frac{\bar{f} - \bar{l}}{1 + \bar{a}(1-\tau)} \qquad L^* = \frac{\bar{a}\bar{f}(1-\tau) + \bar{l}}{1 + \bar{a}(1-\tau)}$$
>
> Of course, you could as well do the same calculations again. Therefore, the tax τ increases the wage paid by the employer by the following amount (of course, what the employee gets $w^*(1-\tau)$ decreases):
>
> $$\frac{\partial w^*}{\partial \tau} = \bar{a}\frac{\bar{f} - \bar{l}}{\left[1 + \bar{a}(1-\tau)\right]^2} > 0.$$

The tax leads to the destruction of jobs:

$$\frac{\partial L^*}{\partial \tau} = -\bar{a}\frac{\bar{f} - \bar{l}}{[1 + \bar{a}(1 - \tau)]^2} < 0.$$

Exercice 2 (20 points)

5. (20 points) Consider the following Solow model. Output Y_t is produced using capital K_t and labor L_t according to the production function: $Y_t = \bar{A}K_t^{1/4}L_t^{3/4}$. (note the exponents !) The amount of labor is fixed $L_t = \bar{L}$, and a fraction of $\bar{s}$ of output is saved by agents in this economy, so that investment is $I_t = \bar{s}Y_t$. Capital depreciates every period at a rate $\bar{d}$.

   (a) (4 points) Write the law of motion for capital. Solve for the steady state value of capital K^*, and the steady state value of Y^*, in this economy.

   Solution:

   $$\begin{aligned} \Delta K_{t+1} = K_{t+1} - K_t &= \bar{s}Y_t - \bar{d}K_t \\ &= \bar{s}\bar{A}K_t^{1/4}\bar{L}^{3/4} - \bar{d}K_t \end{aligned}$$

   Steady-state obtains with $\Delta K_{t+1} = 0$, so that:

   $$\bar{s}Y^* = \bar{d}K^*.$$

   The production function gives a second equation:

   $$\bar{s}Y^* = \bar{s}\bar{A}\left(K^*\right)^{1/4}\bar{L}^{3/4}.$$

   Again, this is a two equations/two variables system whose solution is:

   $$K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{4/3}\bar{L} \qquad Y^* = \left(\frac{\bar{s}}{\bar{d}}\right)^{1/3}\left(\bar{A}\right)^{4/3}\bar{L}.$$

   (b) (4 points) Comment the dependence of Y^* with $\bar{A}$. Compare with the production model. Why is there a difference?

   Solution: There is an exponent 4/3. The production function has linearity in total factor productivity. The reason is that capital accumulation amplifies productivity differences, because more productive countries accumulate more capital, which increases their production further.

(c) (4 points) Imagine that the government can enact policies aimed at targeting people's savings rate $\bar{s}$. Which savings rate would maximize steady-state production ? What would steady-state consumption then be equal to?

> **Solution:** Y^* is maximized when savings are maximum, so the savings rate maximizing steady state production would be $\bar{s} = 1$. However people would be starving in this economy. All its output would be used towards increased capital accumulation.

(d) (4 points) Using that the steady state value for consumption is equal to $C^* = (1 - \bar{s})Y^*$, show that the savings rate $\bar{s}$ has two opposing effects on consumption. Give an intuition.

> **Solution:** $(1 - \bar{s})$ decreases in $\bar{s}$ while Y^* increases in $\bar{s}$. On the one hand, steady state consumption increases when savings increase because there is then more production. But on the other, consumption decreases because a higher fraction of production is then saved rather than consumed.

(e) (4 points) Adam Smith once wrote: "Consumption is the sole end and purpose of all production". Calculate the steady-state consumption maximizing level of the savings rate $\bar{s}$ in the Solow model.

> **Solution:** You need to maximize the following:
>
> $$C^* = (1 - \bar{s}) \left(\frac{\bar{s}}{\bar{d}}\right)^{1/3} \left(\bar{A}\right)^{4/3} \bar{L}.$$
>
> The first order condition gives (note you actually just need to maximize $(1 - \bar{s})(\bar{s})^{1/3}$):
>
> $$\frac{\partial C^*}{\partial \bar{s}} = 0 \quad \Rightarrow \quad -\bar{s}^{1/3} + \frac{1}{3}\bar{s}^{-2/3}(1 - \bar{s}) = 0 \quad \Rightarrow \quad \boxed{\bar{s} = \frac{1}{4} = 25\%}.$$

Exercice 3 (30 points + 6 bonus points)

6. (36 points) We consider an economy with the following production function: $Y_t = \bar{A}K_t^{1/3}L_t^{2/3}N_t$, where the amount of land N_t is also entering as an input in the production function. (firms need land to produce, for example because they need offices) Assume that the number of people working is fixed and given by: $\bar{L}$. Capital depreciates at rate $\bar{d}$, and the savings rate is constant equal to $\bar{s}$. It is also assumed that there is a fixed quantity of land in the economy given by $\bar{N}$.

   (a) (6 points) Write the law of motion for capital, as a function of Y_t and K_t.

Solution:

$$\Delta K_{t+1} = K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t.$$

(b) (6 points) Replace Y_t in the previous expression, to derive the law of motion for the capital stock, where only the capital stock and exogenous variables appear.

Solution:

$$\Delta K_{t+1} = \bar{s}\bar{A}\bar{L}^{2/3}\bar{N}K_t^{1/3} - \bar{d}K_t.$$

(c) (6 points) Explain, using a graph as well as words why the level of capital converges to a steady-state. Take the example of an initially lower than steady state amount of capital $K_0 < K^*$. (*Hint:* note that the introduction of land does not fundamentally change the capital accumulation process, as land is assumed to be in constant supply.)

Solution: See the TA section.

Solution: Same as Solow, replacing the constant $\bar{A}\bar{L}^{2/3}$ by $\bar{A}\bar{L}^{2/3}\bar{N}$.

(d) (6 points) What is the marginal product of land?

Solution: The marginal product of land is given by:

$$\frac{\partial Y_t}{\partial N_t} = \bar{A}\bar{L}^{2/3}K_t^{1/3}.$$

(e) (6 points) Imagine the economy starts with an initially too low level of capital (relative to the steady state level). How does the marginal product of land varies over time? Why?

Solution: As the capital stock increases, the marginal product of land increases from the previous equation. This is intuitive: land being in fixed supply, its scarcity is becoming all the more limiting as the economy produces a lot. This helps explain why the price of land in practice covaries with GDP. (in booms, house prices are high, while they are low in recessions) It also helps explain why China has had soaring real estate prices recently as its economy was accumulating more capital, or as it was becoming more wealthy in general.

(f) (6 points) **Bonus:** extend this model to one where there is a research sector, a production sector, and ideas accumulate over time. Go as far as you can in that investigation. (if needed, use the back of the sheet !)

Solution: Again, see the notes posted on the Website, except you replace the constant $\bar{A}\bar{L}^{2/3}$ by $\bar{A}\bar{L}^{2/3}\bar{N}$.